

Understanding gradients from an EM perspective

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Introduction and summary

This article contains some of the electro-magnetic background information, the MR physicist may find useful in understanding the capabilities and limitations of practical MRI scanners. It is not about designing gradient coil hardware but since the properties of practical systems depend very much on the way the coils are built, it does contain some information about the coils as well. First the basic properties of magnetic field gradients will be described, including the field components in the directions perpendicular to the main field. This will first be done for an ideal case of perfectly uniform gradients and then for more practical MR-like geometries. It will be shown that practical coils are a compromise between desired linearity and practical limitations like dimensions of the coil and available power to drive the coil. Then the interaction with the magnet and other electrically conducting structures, in which eddy-currents can be induced, will be discussed. Finally, the interaction with the patient, in particular peripheral nerve stimulation, will be briefly described.

Gradient magnetic fields

MR scanners usually feature a system of three independently controllable coils for superimposing gradient fields on the homogeneous background field. Ideally, the scanner should be able to generate fields inside the imaging volume described by the simple equation:

$$B_z(x,y,z,t) = G_x(t) x + G_y(t) y + G_z(t) z$$

Where B_z is the field component in the direction of the main field, $G_i(t)$ is the time-dependent gradient strength in spatial direction i and x , y , and z is the distance from the center of the imaging volume (iso-center) along the three coordinate axes. Gradient waveforms $G_i(t)$ have a frequency content up to a few kilohertz, so from EM point of view, these fields are quasi-static. This implies that gradient fields inside the bore of the system are solutions of the Laplace equation, all fields are nicely in-phase with the currents generating them and we do not need to worry about these fields being affected by interaction with the patient (the effect of these fields on the patient is a separate issue).

A very important property of quasi-static magnetic fields is that magnetic flux is continuous. Mathematically, this can be formulated as:

$$dB_x/dx + dB_y/dy + dB_z/dz = 0$$

For gradient fields, this means that a gradient in B_z always causes a gradient in another field component. These so-called concomitant or Maxwell gradients can cause phase errors in some MR imaging sequences. The continuity of flux also implies that the fields of the coil do not stop at the boundary of the coils generating them but have to turn around on the outside of that coil and potentially induce eddy-currents in conducting magnet parts further outside.

Maxwell gradients

The magnitude of the gradient fields is typically of the order of 1% of the main field strength. The field the protons see is the modulus of the sum of the field generated by the gradient coils and that of the main magnet. For the majority of imaging sequences the effect of B_x or B_y gradient fields can be neglected. In those cases where the transverse fields do become relevant, their effect is that they always increase the length of the field vector. If gradients are reversed, the effect on $|B|$ remains additive and as a result, the Maxwell fields cause a gradually increasing phase advance of spins in regions where the transverse field is larger. As these transverse field components are inherently there, there it is not possible to design coils that do not have them.

Magnetic stored energy

Every magnetic field distribution is associated with a magnetic energy. This energy is the volume integral of $B \cdot H$ over the entire volume where the coil makes field (both inside and outside). It is also equal to $E = \frac{1}{2} L I^2$ of the coil (L being the inductance and I the operating current). Although L and I are coil-specific parameters, the field energy only depends on the field distribution, and is independent of the turns distribution generating this field. The stored energy determines how much power has to be supplied or absorbed by the gradient power supply to build up this field or to get it back to zero. For a linear ramp of the gradient field with risetime t_{ramp} , the peak power P_{max} the amplifier has to deliver is approximately:

$$P_{\text{max}} = 2 E / t_{\text{ramp}}$$

For typical gradient field strengths (~ 10 mT/m) a whole-body MR gradient coil has a stored energy of ~ 10 Joule. For a 0.2 ms ramp, the amplifier has to supply ~ 100 kW. The stored energy of a gradient field scales with the square of the gradient strength. For a cylindrical coil the field energy is also proportional to the fifth power of its diameter, so increasing patient bore size is very expensive in terms of required gradient power.

Current distributions generating ideal gradient fields

Since a pure linear gradient is a solution of the governing Laplace equation, there is no fundamental reason why gradient fields cannot be perfect in terms of linearity. All it takes to generate such a perfect gradient is a matching current distribution on the surfaces bounding the volume in which the field is desired. Figure 1 shows feasible field distributions inside a cylindrical surface with completely closed ends, where currents are allowed over the entire surface. This is of course totally impractical for an MR scanner, but this exercise nicely illustrates some of the intrinsic properties of gradients mentioned before. Unlike practical coils, where many different current patterns exist, generating essentially the same useful field, the current distribution for an ideal gradient filling the entire volume inside the conducting boundary is unique.

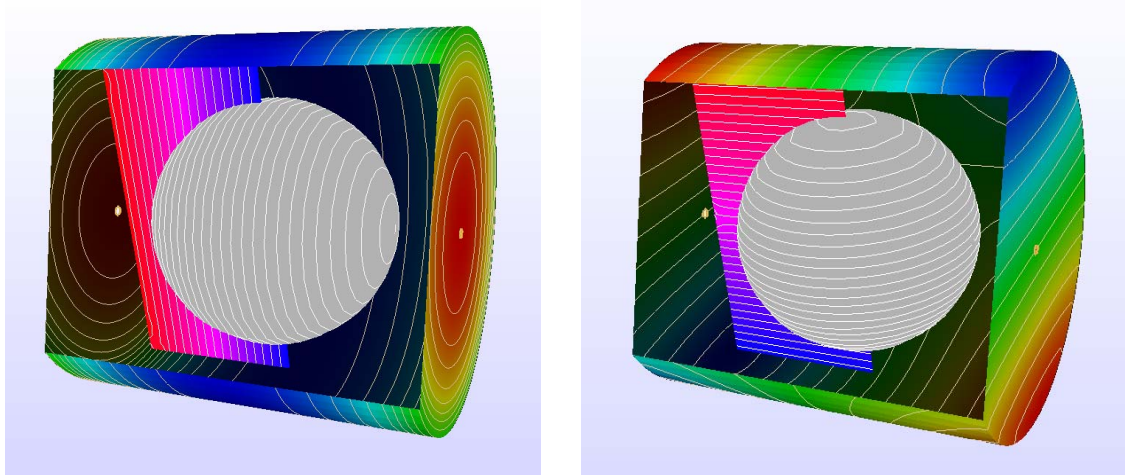


Figure 1 Ideal gradient fields (left: G_z , right: G_{xy}). Z-component of field shown.

Note that here and in the following figures, the field is characterized by iso-contour lines; for a gradient field, such contour lines are ideally parallel and equally spaced. Such a plot on the surface of the spheroid gives a good impression of the quality of a coil, since the deviations from the ideal field always get smaller if you move towards the iso-center.

Even though the field quality is perfect, the closed coils are extremely efficient in terms of stored energy: at 10 mT/m the z-gradient field in a volume of 750 mm diameter and length has only about 2 Joule and the transverse gradients at the same strength have an energy of only 1.5 J. These numbers constitute lower limits of E which no practical gradient of the same strength and volume can improve upon.

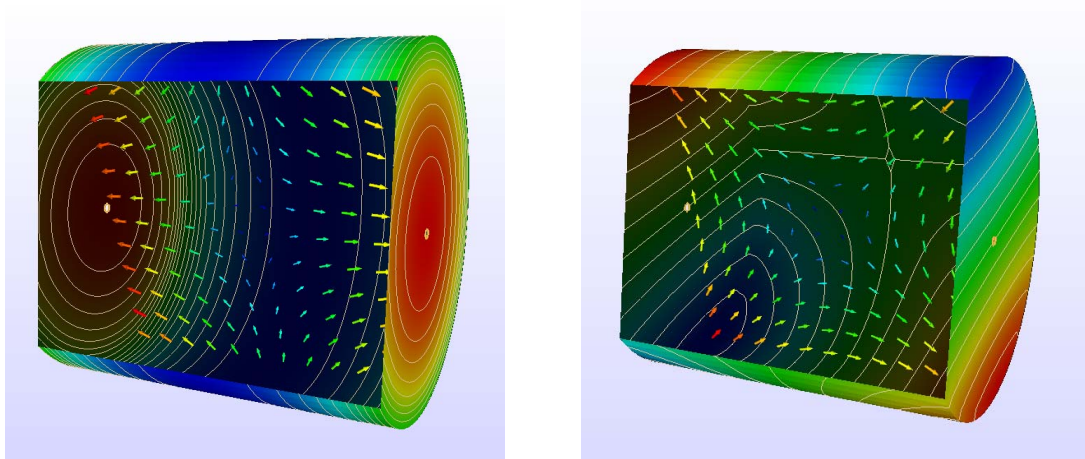


Figure 2 Field vectors of ideal G_z and G_{xy} gradients

Figure 2 shows the two ideal gradient fields in vector form. The z-gradient field has a radial component which increases linearly in magnitude with distance from the z-axis. The strength of the transverse gradient is half of the z-gradient. The transverse gradients have an equally large gradient in the transverse field component (zero in the $z=0$ plane, increasing towards the end of the coil). In the imaging volume these field patterns are very similar to those of practical coils.

Field characteristics of practical gradient coils.

Removing the end caps of the tube carrying the gradient current makes it impossible to create the field boundary conditions needed to make perfect gradient fields. The coil designer now has to find a useful compromise trading linearity and field of view against length of the tube and coil stored energy (i.e. gradient amplifier power). The impact of opening up the tube is most severe for the transverse gradient fields.

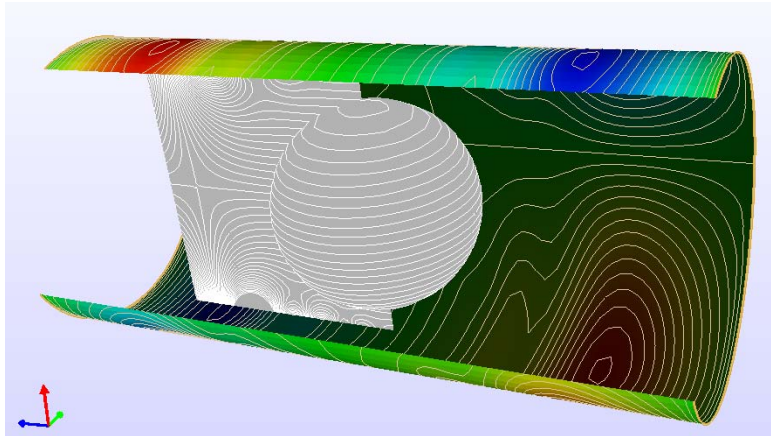


Figure 3 Long transverse gradient coil, very linear

This is well illustrated by the coil shown in Figure 3. This coil has the same diameter as the closed ones shown before, but the cylinder is twice as long. This attempt to achieve approximately the same field quality in the same volume as the ideal coil made the coil's stored energy go up to 12 Joule. Opening up the coil and truncating it also causes higher-order gradients in the field (the lobes seen close to the inner boundary of the tube). These are similar in nature to the field lobes seen in the field of the main magnet. Also shown in Figure 3 are a number of saddle points in the field: these are regions where the field no longer increases monotonically when going away from the center. Such saddle points destroy the one to one relation between field and position and may cause backfolding artifacts in images. The field of view of the coil of Figure 3 is much larger than useful for a practical MRI system. Making the coil shorter and relaxing the requirements on field linearity greatly improves the efficiency and patient-friendliness of a coil, as illustrated in Figure 4.

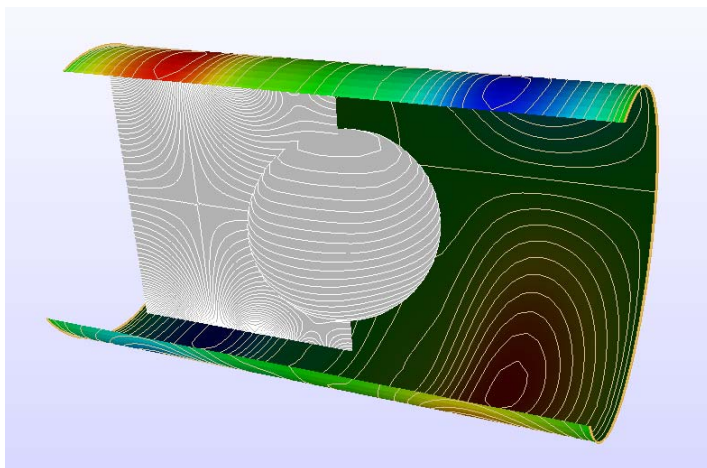


Figure 4 shorter transverse gradient coil, compromised linearity

External gradient field and eddy-currents

As was already mentioned in an earlier section, the magnetic field of a gradient coil does not stop at the boundary of the coil. The figure below shows the same gradient coil as the one in Figure 4, but with the field in vector form.

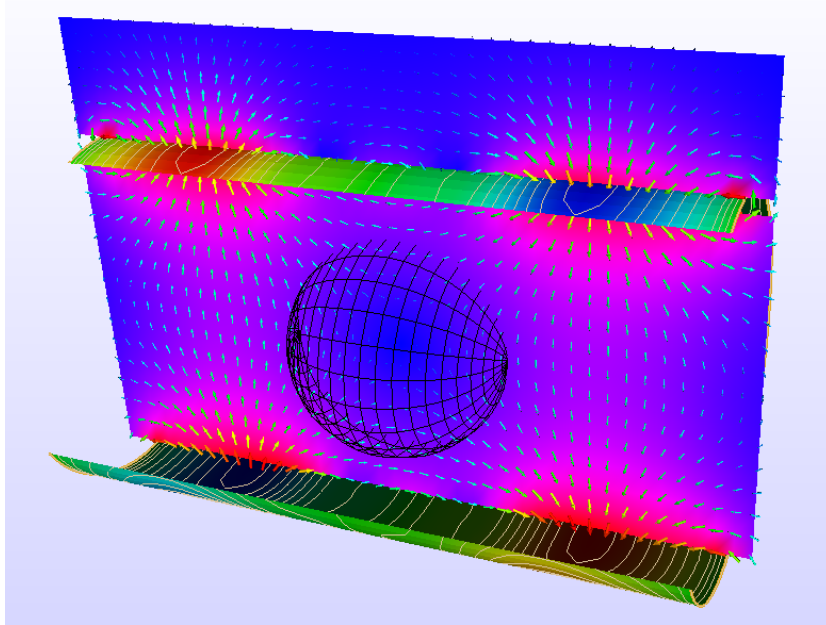


Figure 5 Field vectors of transverse gradient coil, unshielded. Colors: modulus of B

The field just outside the conducting cylinder is just as large as it is just inside it. If this coil is inserted into a magnet with conducting cylinders for vacuum container, radiation shield and coil support, the external field will induce large eddy-currents, which will counteract the useful gradient field. State of the art gradient coils are actively shielded: the primary coil is surrounded by a shield coil, designed such that all magnetic flux escaping from the primary coil is forced through the annular gap between primary and shield coils.

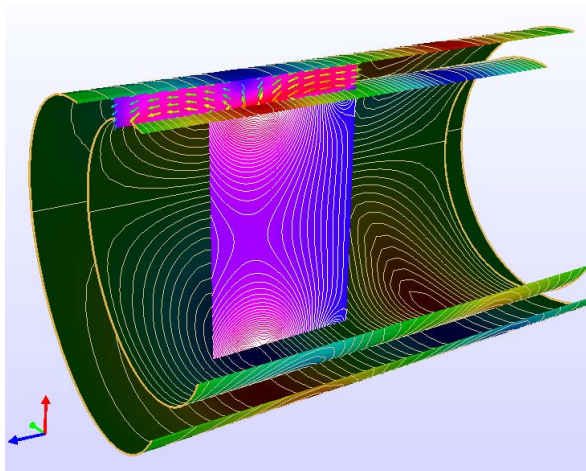


Figure 6 Return flux in actively shielded transverse gradient coil. Map inside coil shows x-component of field

Active shielding again causes the stored energy of the coil to go up, typically by a factor 2 compared to the primary coil in free space. The more the external field is compressed, the higher the energy and the power needed to drive the coil. In principle, it is possible to design a continuous current distribution on the shield part of the coil such that there is virtually no external field (except for end-effects). In practice, however, there is inevitably always some field left, mainly due to the fact that the current distribution has to be discretized. This remaining external field will induce eddy currents in a complicated pattern, as illustrated in Figure 7.

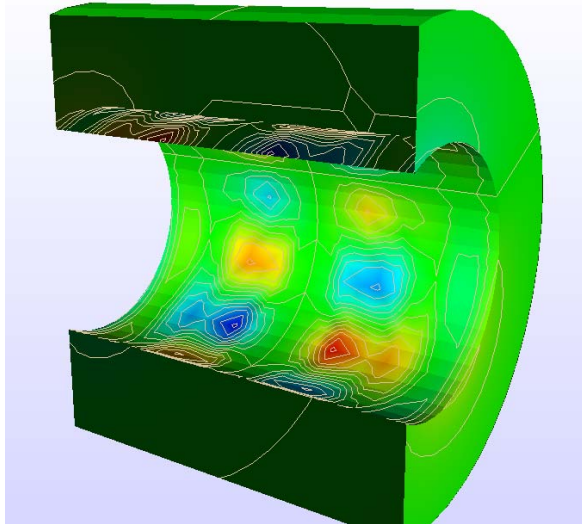


Figure 7 Typical eddy-current pattern due to residual external field

The field due to these eddy-currents is by no means a field of the same shape as the field generated by the coil itself. It will have components with the same symmetry as the gradient field, but which increase in magnitude with the 3rd, 5th or even higher powers of the distance from the iso-centre. Furthermore, due to manufacturing imperfections, there will also be external fields with other symmetries. And finally, eddy currents, which generate field, will also be induced in the conductors of the coil itself.

The eddy-currents will decay with time constants depending on where they were induced and also depending on the pattern of the induced currents. Currents induced in conductors at room temperature typically have time constants of the order of 1 ms or smaller. Eddy currents in the radiation shield of the magnet decay with a characteristic time of typically 20 milliseconds. And if the coil would couple to well conducting structures inside the superconducting magnet, characteristic decay times of seconds can be expected.

Due to the eddy-current fields from all sources described above, the simple relation for the gradient field as the product of a time-dependent part and a space-dependent part is no longer valid.

Peripheral nerve stimulation hazard

The effect of the gradient fields on the patient is that the changing magnetic fields induce electric fields in the patient's body. The resulting voltages must be kept small enough to avoid nerve stimulation (painful) or cardiac arrest. The voltages induced in the patient are mainly due to the

component of the gradient fields normal to the body. When a transvers gradient is excited, the largest normal field occurs there where the z-component of the gradient field is zero, typically 100 mm away from the boundary of the imaging volume in axial direction. The radial component of the z-gradient field is, to first approximation, independent of the axial position. The exact prediction of where the highest voltages will occur and how large they will be requires a full electromagnetic model of the patient, including the distribution of the electrical conductivity. In practice, the manufacturer of the scanner has limited the parameter space of the system so as to ensure that the safety of the patient cannot be jeopardized.