MR Physics for Physicists: Electromagnetic Fields in MRI: From Theory to Practice

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TAKE-HOME MESSAGES

- Any material that is exposed to an external static magnetic field becomes magnetized. This is the source of a field perturbation also referred to as *demagnetization field.*
- Gradient echo phase images reflect the magnetic field at the sites of the nuclei.
- The Lorentz concept facilitates calculation of the magnetic field at the site of a nucleus.
- A theoretical description of the relation between the physical quantity *magnetic susceptibility* and gradient echo phase MRI allows quantifying the magnetic susceptibility distribution of tissue.

TITLE: Static magnetic field: magnetic field (in)homogeneity, susceptibility-related contrast & artifacts

TARGET AUDIENCE: Ph.D. candidates, graduates and scientists, who are interested in understanding the relations between Maxwell's equations, magnetic susceptibility, magnetic field inhomogeneities and imaging artifacts, and how these relations can be used to infer from the MRI signal phase on physical tissue properties.

OBJECTIVES: This course will derive from first principles the relation between magnetic tissue properties and the measured MRI signal. Upon completion of this course, participants will be able to explain why a homogeneous static main magnetic field is important for MRI, employ the relation between local magnetic fields and measured MRI signal, and understand how tissue magnetic properties and magnetic architecture can be derived from MRI phase images *in vivo*.

STATIC MAGNETIC FIELD AND DEMAGNETIZATION FIELD: The presence of a strong magnetic field represents the fundamental basis of the nuclear magnetic resonance effect. In modern MRI scanners, the static main magnetic field is relatively homogeneous. Magnetic field inhomogeneities, depending on their strength and on MR imaging parameters, can cause image distortions, signal attenuation through intra-voxel dephasing, and variations of the voxel mean magnetic field that can be observed, e.g., in the phase of gradient echo images (see below).

Any material that is exposed to an external static magnetic field, such as that in an MRI scanner, is magnetized, which is the source of an additional field, the so called *demagnetization field*[1]. The demagnetization field is the field associated with the moving charges in the atoms or molecules constituting the sample. Both the orbital motion of electrons (localized current density) and the intrinsic moment of electrons (spin) cause a magnetic field distribution around each atom or molecule[2]. The demagnetization field ultimately depends on the strength and direction of the applied external field and on sample composition and geometry.

THEORETICAL DESCRIPTION OF THE MAGNETIC FIELD (LORENTZ APPROACH): Calculation of the demagnetization field requires carrying out the summation over all magnetic moments in the sample, which may prove very difficult for a macroscopic sample. A simple *macroscopic* averaging procedure cannot be used to calculate the field in the context of MRI, because the nuclear spins used for the MR experiment (usually hydrogen nuclei) are *microscopic* field probes. Consequently, in the context of MR, although measurement volumes are generally mac-

roscopic (μm to mm scale), the *microscopic* magnetic environment of the *nuclear* spins must be taken into account when calculating the magnetic field acting on a *nuclear* magnetic dipole moment. *Macroscopic* derivations that account for magnetic moments only in a spatially averaged form as bulk macroscopic average quantities cannot be used directly, because the MR measurement does not involve such a spatial averaging process over the microscopic magnetic fields.

It was noted by Lorentz more than 100 years ago[3] that calculation of the total field of all dipoles can be simplified by virtually separating the environment surrounding the location into a *near* region, inside which magnetic moments are considered as discrete entities with individual dipole fields, and a *distant* region, in which they are treated mathematically as a continuous magnetic moment density. The virtual surface separating the near and distant regions is called *Lorentz surface*. The Lorentz surface may be defined almost arbitrarily because it is only a mathematical concept to facilitate the field calculations and it has no deeper physical foundation[3,4]. However, contrary to common misconception, the Lorentz surface cannot be chosen arbitrarily small. The surface must be defined in such a way that the total field at the location of the nucleus resulting from all dipoles in the distant region can be modelled as resulting from a continuous medium. Lorentz recommended the surface to be larger than the mutual distance of nearby adjacent magnetic moments[3,4]. Loosely speaking, the volume enclosed by the surface should be microscopically large, but macroscopically small. For simplicity, it is common to choose a spherical Lorentz surface, also referred to as the *Lorentz sphere* (LS)[3,5-8].

Independent of the chosen surface, the *Lorentz approach* turns the summation over moments in the distant region into an integration over a continuous, macroscopic quantity, and allows to separately calculate the field contributions corresponding to moments in the near and distant regions, respectively[4], yielding

$$\vec{B}(\vec{r}) = \vec{B}_0(\vec{r}) + \mu_0 \cdot \int_{V_\mathrm{d}(\vec{r})} \left\langle \sum_{j\in K_\mathrm{d}} \frac{3\hat{l}(\hat{m}_j \cdot \hat{l}) - \hat{m}_j}{4\pi \cdot \| \vec{r} - \vec{r}' \|_2^3} \cdot \left\| \vec{m}_j \right\|_2 \cdot \delta(\vec{r}' - \vec{r}_j) \right\rangle \mathrm{d}^3\vec{r}' + \underbrace{\sum_{j\in K_\mathrm{n}} \vec{b}_d(\vec{r} - \vec{r}_j, \vec{m}_j)}_{\equiv \vec{B}_\mathrm{near}(\vec{r})},$$

where B_0 is the applied external (main) magnetic field, V_d is the distant region, K_d and K_n are the sets of indices j corresponding to magnetic moments $\vec{m}_j$ in V_n, and V_n, respectively, $\hat{l}$ and $\hat{m}_j$ are the unit vectors in the direction of $\vec{r} - \vec{r}'$ and $\vec{m}_j$, respectively, and b_d is the dipole field. The symbol $\langle \cdot \rangle$ indicates a suitable spatial-temporal averaging procedure[9]. The audience is referred to the work by Russakoff for an in-depth discussion of this averaging procedure[10].

In the MRI literature it is usually assumed that magnetic moments are randomly distributed in the sample. In this case, the field contributions from all dipoles *within* the Lorentz sphere (the right most term in the equation above) average to zero[4]. However, this assumption may not be valid in (diffusion restricted) biological tissues, such as brain white matter, where the magnetic architecture is known to be anisotropic[11,12]. In this case, the near field depends on the local microscopic arrangement of dipoles and can, hence, be spatially dependent and different from zero.

MAGNETIC SUSCEPTIBILITY: The magnetic susceptibility χ is a dimensionless macroscopic physical property that characterizes the dependence of the (induced) magnetization on the local magnetic field. In the most general case, χ is a second-order (rank-2) tensor that depends on

the local magnetic field and magnetic history (hysteresis)[13]. Materials with negative susceptibility χ are called diamagnetic materials; whereas the susceptibilities of paramagnetic and ferromagnetic materials are positive. Most biomaterials are diamagnetic or weakly paramagnetic with susceptibilities on the order of $\pm 10^{-5}$ to $\pm 10^{-6}$ ($|\chi|<<1$)[14]. Susceptibilities of ferromagnetic materials are several orders of magnitude higher ($\chi > 1$); these materials are generally considered MRI unsafe. For non-ferromagnetic materials the total field can be written in a first-order approximation as a convolution integral[1,8,15,[16:

$$B\ (\vec{r}) = B_0(\vec{r}) \cdot \int_{V_d(\vec{r})} \chi_{app}(\vec{r}') \cdot b_\chi(\vec{r} - \vec{r}') \mathrm{d}^3\vec{r}',$$

where $b_\chi(\vec{r}) = \frac{3\cos^2\theta - 1}{4\pi \cdot \|\vec{r}\|_2^3}, \vec{r} \neq 0$ is a *macroscopic* unit-dipole function or, mathematically speaking, the Green's function of the inverse macroscopic field-to-source problem[1] (θ is the angle between $\vec{r}$ and the applied magnetic field). The derivation of this equation implicitly assumes that the Lorentz near field equals zero.

Ever since the early days of magnetic resonance imaging (MRI) strong interest existed in the tomographic quantification of magnetic susceptibility[15,17-19], because it was noticed that tissue magnetic susceptibilities differ depending on tissue chemical composition[20,21]. Today we know that magnetic susceptibility of brain tissue is dominated essentially by four major constituents: water, myelin, iron and calcium[22-24]. Myelin, the lipoprotein sheath surrounding axons, is more diamagnetic than water[11,12,22,25], potentially allowing to pick up tissue demyelination associated with neurodegenerative diseases[26]. Depending on its chemical form iron can have a relatively high magnetic susceptibility[25,27,28]. Calcium, like myelin, has a lower (more diamagnetic) susceptibility than water, potentially enabling differentiation between calcium and blood (heme iron) or blood products in brain lesions[29-31].

PHASE IMAGING: Due to the Larmor relation the MR signal frequency directly depends on the magnetic field at the site of the nucleus[32]:

$$f_L(\vec{r}) = -\frac{\gamma}{2\pi} \cdot B(\vec{r})$$

where γ is the gyromagnetic ratio. MR measures the Larmor frequency referenced to a technical, pre-adjusted demodulation frequency f_0, which transforms the frequency from the laboratory frame to a rotating frame. Gradient echo (GRE) MRI[33,34] can be understood as sampling at one or more time points, $t = TE$ (echo time), the complex-valued free induction decay (FID) MR signal, which follows the MR radio-frequency excitation[12]. The GRE signal in an imaging voxel V can be approximated as

$$I(V,TE) \approx \int_V a(\vec{r},TE) \cdot e^{-i \cdot \int_0^{TE} (f_L(\vec{r}(t')) - f_0) \mathrm{d}t'} \mathrm{d}\vec{r}^3,$$

where $a(\boldsymbol{r},\mathrm{TE})$ indicates the amplitude of the transverse *nuclear* magnetization at location $\vec{r}$ and time TE. If the magnetic susceptibility is homogeneous and constant within each imaging voxel, the phase φ of the complex-valued GRE-signal, $I(V,TE)$, yields for each voxel estimates of f_L of

all spins in that voxel[32] (right-handed MR system[35]): $\varphi(TE) = \varphi^0 + 2\pi \cdot (f_L - f_0) \cdot TE$. The time-independent phase term φ^0 represents the signal phase at echo time of 0 ms, usually defined by the end of the excitation radio-frequency (RF) pulse. Gradient echo phase images have been demonstrated to provide unique anatomical contrast complementary to other MRI techniques, in particular complementary to gradient echo *magnitude* images[36-38]. The phase contrast is a direct result of the relation between gradient echo phase, demagnetization field and underlying magnetic susceptibility distribution (see above).

MAGNETIC SUSCEPTIBILITY MAPPING: Quantitative susceptibility mapping (QSM) is a post-processing technique for gradient echo phase images that retrieves quantitative information about the underlying magnetic susceptibility distribution of a sample or biological object[1,24,25,39-43]. QSM involved several sophisticated processing steps, including estimating the magnetic field distribution from raw MRI phase data, eliminating so-called *background field* contributions that may result from magnetization induced outside of the imaging field-of-view in MR-invisible areas, and, finally, solving the inverse problem from field perturbation to magnetic susceptibility. Each processing steps needs to be carried out with utmost rigor because the final inversion step is highly sensitive to noise and errors in the input field pattern[44,45]. Various methods have been developed in the recent past to address the challenging problem of QSM and several groups have presented maps of apparent bulk magnetic susceptibility and even maps of magnetic susceptibility tensors with high image quality and unprecedented anatomical detail.

LIMITATIONS: Although the field of quantitative phase imaging has experienced an impressive growth and achieved substantial scientific progress over the past several years, several challenges remain. Most troubling is that microstructural near-field effects have been neglected so far in QSM; the fundamental basis of current QSM algoriths is that the Lorentz near field vanishes, which is questionable in anisotropic tissue, such as the brain. In fact, the Lorentz effect has recently been hypothesized as a cause of phase image contrast itself[46]. The situation becomes even more complex when multiple tissue compartments are considered with different dipole distributions and different T_1 and T_2^* relaxation properties (as in brain tissues)[12]. Yablonskiy and Sukstanskii recently presented a comprehensive theoretical description of these effects, the Generalized Lorentzian Tensor Approach (GLTA)[47]. However, it is unknown so far how the inverse GLTA problem can be solved, i.e. how the magnetic susceptibility distribution can be calculated accounting for phase effects associated to the microscopic magnetic architecture.

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