Maxwell Equations & EM Field Modeling for MRI

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Learning Objectives:

- Maxwell's equations and their relation to fundamental principles in MR
- Characteristics of electromagnetic fields in low and high-field MR systems
- Basics of numerical methods for solving Maxwell's equations
- Pros and cons of selected numerical methods with respect to typical MR-related problems

Target audience: MR physicists, scientists, and engineers with an interest in understanding the characteristics of electromagnetic fields involved in MR and in understanding numerical methods for solving Maxwell's equations to analyze MR-related problems with respect to hardware design and safety.

Principles: In MR systems, fields from different bands in the electromagnetic spectrum are utilized to manipulate magnetic moments of nuclei as well as to detect the MR signal. Thus, the static magnetic field B_0 polarizes spin ensembles and switched magnetic field gradients (G_x, G_y, G_z) at frequencies up to 10 kHz are applied for spatial localization. Further, radio frequency (RF) transmit coils generate fields at the Larmor frequency for spin excitation, whereas RF receive coils detect the MR signal. The spatial field distributions in the different frequency ranges obey Maxwell's equations (MWE). To study Maxwell's equations is therefore of interest to understand fundamental principles in MR and to understand implications on spin excitation, signal reception, and hardware design that arise from the characteristics of electromagnetic fields in MR systems.

MWEs describe the interrelationship between electric and magnetic fields, electric currents, and electric charges for a given material and source distribution as well as for given boundary conditions. Although analytical solutions of Maxwell's equations for realistic MR-related problems are unavailable, the set of equations can be used to study basic principles and field characteristics in MR systems. For this purpose, it is useful to take a look at MWEs specified for the magnetostatic (Eq. 1) and quasi-stationary approximation (Eq. 2) as well as for the electromagnetic regime (Eq. 3).

	Magnetostatics	**Quasi-stationary fields**	**Electromagnetic fields**
$\oint_C \vec{H}\cdot d\vec{l} =$	$\iint_S \vec{J}\cdot d\vec{s}$ [1.1]	$\iint_S \vec{J}\cdot d\vec{s}$ [2.1]	$\iint_S \left(\vec{J}+\frac{\partial}{\partial t}\vec{D}\right)\cdot d\vec{s}$ [3.1]
	Ampère's law		
$\oint_C \vec{E}\cdot d\vec{l} =$	–	$-\frac{\partial}{\partial t}\iint_S \vec{B}\cdot d\vec{s}$ [2.2]	$-\frac{\partial}{\partial t}\iint_S \vec{B}\cdot d\vec{s}$ [3.2]
		Faraday's law of induction	
$\oiint_S \vec{D}\cdot d\vec{s} =$	–	$\iiint_V q\, dv$ [2.3]	$\iiint_V q\, dv$ [3.3]
		Gauss's law	
$\oiint_S \vec{B}\cdot d\vec{s} =$	0 [1.2]	0 [2.4]	0 [3.4]
	Gauss's law for magnetism		

The interaction between materials (dielectrics, magnetics, and conductors) and the electromagnetic fields is taken into account by the constitutive equations given here in the frequency domain (in the time domain respective products convert into convolutions) [1]

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(1 + \chi_m)\vec{H} = \mu_0\mu_r\vec{H} \quad [4.1]$$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon_0\vec{E} + \varepsilon_0\chi_e\vec{E} = \varepsilon_0(1 + \chi_e)\vec{E} = \varepsilon_0\varepsilon_r\vec{E} \quad [4.2]$$

$$\vec{J} = \sigma\vec{E} \quad [4.3]$$

The magnetostatic distribution B_0 is determined by Ampère's law (1.1), Gauss's law for magnetism (1.2), and the corresponding constitutive relation (4.1). Biot-Savart's law, which is derived from this set of equations, can be used to calculate B_0 from the steady current flowing in the magnet's windings. Further, by use of (1.2) and (4.1) the effect of media with varying magnetic susceptibility on the B_0 distribution can be determined.

For time-dependent problems, Faraday's law of induction (2.2/3.2) and Gauss's law (2.3/3.3) come into play. Faraday's law gives the relation between the rate of change of the magnetic flux through a surface S enclosed by a contour C and the electric field along the contour. On the one hand this principle is utilized for the detection of the MR signal, as the time-dependent transversal component of the net nuclear magnetization induces a voltage/current in a properly aligned receive coil. On the other hand, a detrimental effect due to Faraday's law occurs during spin excitation when the magnetic field of the RF transmit coil induces an electric field inside the lossy human body tissue. The corresponding energy deposition in the human body is a major safety concern and, consequently, needs to be determined and monitored by proper methods.

The difference between the time-dependent representations of MWE in (2) and (3) is that in the electromagnetic regime (3) an extra term has been introduced into Ampère's law (2.1) that takes into account effects from the so-called displacement current density $\frac{\partial}{\partial t}\vec{D}$. Maxwell introduced this term and modified (2.1) in such a way that the circulating magnetic field in a closed loop is now related to the electric current as well as the electric displacement current passing through the loop. So, the electric displacement current density acts similar to the electric current density inside conductors. Consequently, the displacement current density in (3.1) allows under certain conditions that EM fields detach from sources and propagate in space. That is, an EM wave is formed by electric and magnetic fields interdependently produced by the change in the other type of field. In contrast, in the quasi-stationary regime the fields do not propagate and, hence, do not show a corresponding phase delay; thus, the fields vary in time in phase with the time dependency of the source terms, e.g. currents on gradient and RF coils.

The transition from the quasi-stationary to the electromagnetic regime takes place with increasing electrical size of the problem, which is given by the ratio of wavelength to object size. However, a unique threshold between the two regimes does not exist, and the characteristic of the fields depend on the actual configuration. In a good approximation, fields from gradient coils as well as fields at Larmor frequencies for rather low B_0 can be treated in the quasi-stationary regime, whereas RF fields in whole-body high-field MR systems ($B_0 > \sim 2$ Tesla) have to be treated in the electromagnetic regime, which sometimes has a significant impact on spin excitation, signal reception, and hardware design compared to low-field MR systems.

Analytical solution of Maxwell's equations can only be derived for simplified configurations. To analyze more realistic and more complex scenarios, numerical methods have to be applied to solve Maxwell's equations, in particular if heterogeneous material distributions are to be taken into account, e.g. anatomical body models. Numerical simulations have become an indispensable tool for compliance testing as well as for design optimization of transmit and receive coils of MR systems. Since the entire threedimensional field distribution can be obtained, it is possible to extract various pieces of useful information for realistic exposure scenarios that cannot be obtained from measurements in phantoms or in vivo in comparable detail. In particular, with respect to compliance testing, numerical computation of RF fields in body models is currently the only practical way to obtain realistic SAR distributions as are necessary to guarantee compliance with limits for the localized SAR. A review on numerical computation in MRI including a comprehensive literature survey can be found in [2].

Most commonly, local numerical methods like the Finite-Difference Time-Domain Method (FDTD) [3], the Finite Integration Technique (FIT) [4], and the Finite Element Method (FEM) [5] are used to calculate electromagnetic field distributions in MRI. FDTD and FIT are very robust methods that utilize a hexahedral mesh to discretize the material distribution in the entire solution domain of the problem of interest. Since the memory usage is only linear with the number of mesh cells, FDTD/FIT can solve large-scale problems with moderate memory demands. However, since the field distribution is solved in the time domain, long computation times may be necessary to reach the desired accuracy for field problems that include highly-resonant structures. Further, a simulation run per excitation port is necessary, which increases the computation time for multi-channel RF coils. On the other hand, FDTD codes can handle highly heterogeneous material distributions and can be easily parallelized, so they can make use of current parallel hardware architectures to significantly reduce computation times.

FEM is commonly used with a tetrahedral mesh, offering greater flexibility and accuracy to model curved structures. Again, the entire space needs to be discretized, but typically FEM requires a smaller number of tetrahedra compared to the number of FDTD voxel for the same structure. Another advantage is that the FEM algorithm can be formulated in the frequency domain, which makes it well-suited for resonant structures. However, the memory usage of FEM is significantly higher compared to the FDTD method, which restricts this method to problems of smaller geometrical size. Direct solvers generate a large system of linear equations and require matrix inversion to solve the field problem at a memory demand according to $N^{1.8}$ - N^3 depending on the pre-conditioner used. The computation time is independent of the number of excitation ports.

In addition to local methods, integral equation methods [6] and hybrid methods [7] have also been proposed for field computations with RF coils.

Conclusion: Knowledge of Maxwell's equations is the key to understanding fundamental principles in MR imaging as well as to understanding the different characteristics of electric and magnetic fields applied in low and high-field MR systems. This talk will discuss spin excitation and signal reception by use of Maxwell's equations and will illustrate field distributions for realistic MR-related configurations in the quasistatic and electromagnetic regime. Further, basics of numerical methods for solving Maxwell's equations will be discussed as well as their suitability for typical hardware design and safety validation scenarios in MRI.

References:

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