k-Space

Kevin Koch, PhD
Medical College of Wisconsin
Milwaukee, WI

This talk will cover the fundamentals of k-space and how it relates to MR imaging. The presented summary k-space principles aims to help MR users evaluate the tradeoffs associated with their choice of pulse sequence and prescription parameters. We will begin by describing the basics of Fourier Transformations, k-space, and pulse sequences, followed by a presentation of how k-space parameters directly impact image parameters such as spatial resolution, FOV, and SNR. Several "take-home" principles will be highlighted that should help guide the selection of parameter choices for a given application.

1. Overview of Fourier transformation and k-space

The Fourier Transformation is a mathematical process that decomposes any signal, whether it be a 1-dimensional sound wave or a 2-dimensional image, into its frequency components. For example, consider the sound wave created by playing middle C on the musical scale. If we took the Fourier transform (FT) of this note, we would find that it resonates at a single frequency of 262 Hz, which is a measure of how rapidly the air pressure varies over time. The units of Hz correspond to [cycles/second]. In comparison, if we play a musical chord composed of 3 notes, middle C, E, and G, the FT of this sound wave would reveal 3 distinct resonant frequencies at 262, 330, and 392 Hz, lending richness and texture to the sound. The FT operation works both forwards and backwards and no information is lost in the process; therefore, an inverse Fourier transform could be used to convert the 3 frequencies back into the original sound wave.

Similarly, an image is composed of spatial frequencies, which is a measure of how rapidly the image intensity varies over space. By convention, spatial frequency is represented by the letter "k" and spatial frequency space is referred to as "k- space." The units of k correspond to [cycles/cm]. Lower spatial frequencies characterize the overall intensity and general features of an image, while higher spatial frequencies contain information about edges and sharp features. Knowledge of spatial frequencies is extremely powerful; if you had no information about a particular image other than its k-space components, you could perform an inverse Fourier transformation to reconstruct the complete image.

This brings us to the fundamental underlying principle of MRI: raw MRI data are samples in k-space. That is to say that MRI raw data are samples of the Fourier Transform of the imaged object. By performing an inverse FT operation on the raw data, we can recover the desired image.

2. Pulse sequences and k-space trajectories

How does MRI sample data in k-space? By imposing a small magnetic field gradient along a given direction, MRI causes each point along that direction to accumulate a unique phase that is proportional to position and to the strength and duration of the applied gradient. The receiver coil is sensitive to all points within its field-of-view and adds up contributions from all points. The net effect of this process is that the signal produced by the coil at a given time is a sample of k-space. Once all k-space samples have been acquired, the inverse FT can be performed to reconstruct the image.

Different pulse sequences may acquire the k-space data in a different order. The path taken to sample k-space data is known as the k-space "trajectory." The most common pulse sequences sample k-space one line at a time on a Cartesian grid. However, alternative k-space trajectories such as radial and spiral are also used for specialized applications.

3. k-space determinants of spatial resolution and FOV

The manner in which k-space is sampled affects imaging parameters such as spatial resolution and FOV. Specifically, the spatial resolution of an MR image is determined by how far out into the high spatial frequencies the pulse sequence measures. The field-of-view of an image is inversely proportional to the k-space sampling spacing, Δk. We will discuss the conditions under which spatial aliasing can result and steps to avoid it.

4. SNR and its dependencies

An MR image's signal-to-noise ratio (SNR) depends strongly on the spatial encoding used—specifically the voxel volume and the total A/D acquisition time. We will examine how SNR varies as imaging parameters are changed in order to gain an understanding of inherent imaging tradeoffs.

5. Summary

In MRI, spatial resolution is a very costly commodity in terms of SNR penalty, whereas field-of-view increases are free of SNR penalties. These properties are well explained in k-space.