Reconstruction of Non-Cartesian k-Space Data

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Highlights

- Reconstruction of non-Cartesian k-space data is more challenging since it requires handling non-equidistant samples
- Conventional reconstruction grids the non-equidistant k-space samples to a Cartesian space and then uses an inverse Fast Fourier Transform (FFT)
- Non-Cartesian k-space sampling facilitates the use of compressed sensing due to the inherent presence of incoherent aliasing artifacts even for uniform undersampling

Introduction

Non-Cartesian k-space sampling techniques, such as radial or spiral, are less sensitive to motion and undersampling than their Cartesian counterparts. They were already proposed in the origins of MRI [1], but later displaced by Cartesian techniques, which are more robust to magnetic field inhomogeneities and gradient imperfections and easier to reconstruct. Currently, there is a renewed interest in non-Cartesian imaging, particularly after the introduction of compressed sensing and the need for rapid continuous free-breathing imaging. The first part of this talk will review conventional reconstruction of non-Cartesian k-space space data using gridding, including practical algorithms such as the non-uniform FFT (NUFFT). The second part of the talk will discuss more advanced reconstruction ideas that are renewing the interest in non-Cartesian imaging, such as undersampling and sparse reconstruction and the ability to resolve motion as a different dimension.

Conventional reconstruction: Gridding

The most common method to reconstruct non-Cartesian k-space samples is gridding [2-4], where the acquired data are convolved with a kernel, resampled onto a Cartesian grid and transformed to the image space using a FFT operation. The selection of the convolution kernel affects the number of required k-space points. The Kaiser-Bessel kernel provides minimum oversampling and thus is the preferred one in practical algorithms [2]. Gridding reconstruction requires density compensation to reduce the effects of non-uniform sampling density, which can be done by multiplying the acquired non-Cartesian k-space data with the inverse density function. For certain trajectories, such as radial, there are analytical expressions for the sampling density. Other trajectories require estimation of the sampling density using numerical methods [5]. The final step in gridding reconstruction is to remove the effects of k-space convolution on the image. This can be performed using an apodization of the image by the inverse FFT of the gridding kernel. In summary, gridding reconstruction consists of the following steps: density compensation, convolution and resampling, inverse FFT and apodization.

Advanced reconstruction: Undersampling, sparsity and iterative algorithms

The need for rapid continuous free-breathing imaging and particularly the introduction of compressed sensing has revived the interest in non-Cartesian imaging techniques. Undersampling non-Cartesian k-space trajectories inherently results in highly incoherent aliasing artifacts, even for uniform undersampling schemes, which increases the performance of sparsity-based reconstruction algorithms. In fact, the original compressed sensing paper demonstrated the reconstruction of undersampled radial MRI data [6] and subsequent work further promoted this idea [7-9]. Moreover, the self-navigation properties of non-Cartesian imaging and arbitrary data sorting capabilities of golden-angle acquisition schemes have recently enabled to reconstruct organ motion as a separate dimension, which in addition to compensate for motion blurring artifacts, offers access to previously not available physiological information [10].

These advanced image reconstruction algorithms are iterative and thus require going back and forth between Cartesian image space and non-Cartesian k-space. Inverse gridding can be performed in the reverse order of gridding. The Cartesian image is transformed to Cartesian k-space using a FFT operation, convolved with the gridding kernel and resampled onto the non-Cartesian points. In this case, there is no need for density compensation because the starting image is Cartesian. The NUFFT Matlab package [11] implements the forward and inverse gridding operations and is widely used in the development of iterative reconstruction algorithms for non-Cartesian k-space data.

References

1. Lauterbur PC. Image formation by induced local interactions: Examples employing nuclear magnetic resonance. Nature 1973. 242:190-191.
2. Jackson JI, Meyer CH, Nishimura DG, Macovski A. Selection of a convolution function for Fourier inversion using gridding. IEEE Trans Med Imaging 1991.10(3):473–478.
3. Meyer CH, Hu BS, Nishimura DG, Macovski A. Fast spiral coronary artery imaging. Magn Reson Med. 1992; 28(2):202–13.
4. Beatty PJ, Nishimura DG, Pauly JM. Rapid gridding reconstruction with a minimal oversampling ratio. IEEE Trans Med Imaging 2005;24(6):799–808.
5. Pipe JG, Menon P. Sampling density compensation in MRI: rationale and an iterative numerical solution. Magn Reson Med 1999; 41: 179–86.
6. Candès E, Romberg J, T. T. Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information. IEEE Trans Inf Theory 2006;52:489–509.
7. Block KT, Uecker M, Frahm J. Undersampled radial MRI with multiple coils. Iterative image reconstruction using a total variation constraint. Magn Reson Med 2007;57(6):1086-1098.
8. Lustig M, Pauly JM. SPIRiT: Iterative self-consistent parallel imaging reconstruction from arbitrary k-space. Magn Reson Med. 2010;64(2):457-71.
9. Feng L, Block KT, Grimm R, Chandarana H, Kim S, Xu J, Axel L, Sodickson DK, Otazo R. Golden-Angle Radial Sparse Parallel MRI: Combination of compressed sensing, parallel imaging and golden-angle radial sampling for fast and flexible dynamic volumetric MRI. Magn Reson Med. 2014;72(3):707-17.
10. Feng L, Axel L, Chandarana H, Block KT, Sodickson DK, Otazo R. XD-GRASP: Golden-Angle Radial MRI with Reconstruction of Extra Motion-State Dimensions Using Compressed Sensing. *To appear* in Magn Reson Med. 2015
11. Fessler JA, Sutton BP. Nonuniform fast Fourier transforms using min-max interpolation. IEEE Transactions on Signal Processing, 2003;51(2):560-574.