Improving low-rank plus sparse decomposition of dynamic MRI using short temporal snippets

Esben Plenge[1], Tal Shnitzer[1], and Michael Elad[1]
[1]*Technion - Israel Institute of Technology, Haifa, Israel*

Purpose:

In dynamic MRI there is a fundamental trade-off between temporal and spatial resolution. In recent years, the compressed sensing (CS) framework has enabled great improvements to this trade-off. CS theory asserts that an under-sampled signal can be accurately reconstructed under conditions of sparsity in some domain and incoherence of the under-sampling artifacts in that domain. Popular sparsifying transforms in CS MRI include wavelets, the Fourier transform, DCT, and finite differences, and more recently, non-linear transforms using learned dictionaries. In [1], it was proposed to decompose the spatiotemporal matrix $\boldsymbol{M}$, having the vectorized time frames as its columns, into a low-rank component $\boldsymbol{L}$ and a sparse component $\boldsymbol{S}$. In this work we present a new dictionary-based transform that relies on the sparsity of short temporal *snippets* in the dynamic image sequence. We plug this transform into the L+S optimization scheme of [1] as the sparsifying operator. The snippet-based model is designed specifically to reduce temporal fluctuations present in reconstructions of highly under-sampled radial and spiral sampling trajectories.

Methods:

The L+S optimization problem can be given by: $\min_{L,S} \|\boldsymbol{L}\|_* + \lambda \|\boldsymbol{TS}\|_1 \; s.t. \|\boldsymbol{E}(\boldsymbol{L}+\boldsymbol{S}) - \boldsymbol{d}\|_2^2 < \epsilon$. Here, $\boldsymbol{E}$ is the acquisition operator, usually given by $\boldsymbol{E} = \boldsymbol{UFB}$, where $\boldsymbol{U}$ is the under-sampling operator, $\boldsymbol{F}$ is the Fourier transform, and $\boldsymbol{B}$ contains the coil sensitivity maps. $\boldsymbol{T}$ is a sparsifying transform, $\boldsymbol{d}$ is the sampled data and ϵ is an error threshold. $\|\cdot\|_*$ denotes the nuclear norm. After solution of the problem, the spatiotemporal matrix $\boldsymbol{M}$ is recovered by $\boldsymbol{M} = \boldsymbol{L} + \boldsymbol{S}$.

To incorporate the snippet-based model into the L+S scheme, we modify the above expression as follows: $\min_{L,\alpha} \|\boldsymbol{L}\|_* + \lambda \|\boldsymbol{A}\|_0 \; s.t. \|\boldsymbol{E}(\boldsymbol{L} + \boldsymbol{R}^{\boldsymbol{T}}(\boldsymbol{DA})) - \boldsymbol{d}\|_2^2 < \epsilon$, where $\boldsymbol{E}^{\boldsymbol{H}}$ is the Hermitian transpose of $\boldsymbol{E}$. The dictionary $\boldsymbol{D}$ is trained on short temporal signals, so-called *snippets*, using K-SVD [2], and $\boldsymbol{A}$ is a matrix in which the columns are sparse representations of the snippets. Snippets of size $1 \times 1 \times n_S$ are extracted at all pixel locations in the sparse component estimate $\boldsymbol{S} = \boldsymbol{M} - \boldsymbol{L}$ using the snippet extraction operator $\boldsymbol{R}$, and are replaced by their approximations $\boldsymbol{DA}$ using $\boldsymbol{R}^{\boldsymbol{T}}$.

For validation we created a dynamic phantom by segmenting a reference frame of a free-breathing abdominal 4D MRI, and animating it using the deformations fields obtained by registering all frames to the reference frame (MRI data courtesy of Pascal Spincemaille and Yi Wang, Weill Cornell Medical College, NY) and an *in vivo* DCE-MRI data set of the human abdomen available online together with a Matlab implementation of the L+S algorithm [4]. The *in vivo* data set was a 2D + time image, extracted from a 4D acquisition using a golden-angle radial sampling scheme [3] of a total of 600 spokes each with 384 sampling points [1]. An acquisition was simulated on the phantom by adopting the *in vivo* sampling scheme, yielding a 2D + time image. For the experiments with the *in vivo* and phantom data, 5 and 8 spokes per time frame were used, respectively, thus under-sampling the full k-space of 348×348 of each frame by factors of ~70 and 48. In both experiments, snippet sizes of $n_S = 11$, and compact dictionaries of 22 atoms were used. In the phantom experiments, noise was added before the simulated acquisition. In the reported quantitative experiments, the proposed algorithm was initialized by 20 iterations of ordinary L+S, where $\boldsymbol{T}$ was implemented as a finite difference transform, and followed by 20 iterations using the snippet-based sparsifying transform. For comparison, the ordinary L+S was run for 40 iterations.

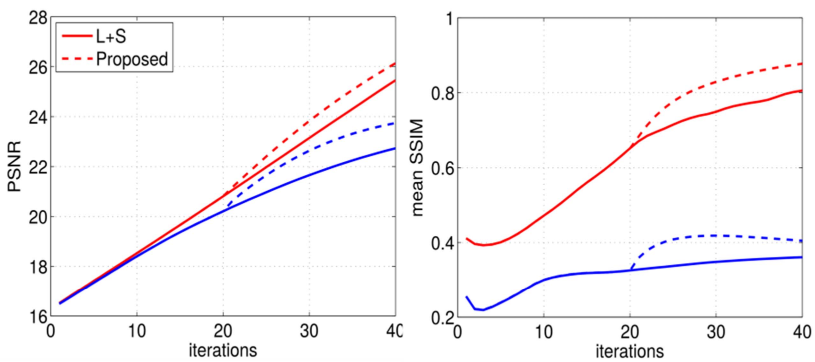

Fig. 1: Quantitative results

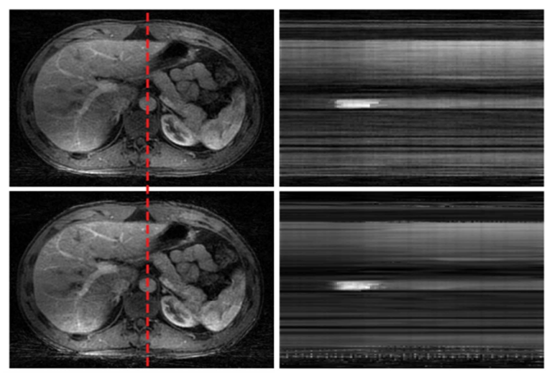

Fig. 2: Qualitative results

Results:

Fig.1: Peak-signal-to-noise ratio (PSNR), and mean structural similarity index (mean SSIM) of reconstructed phantom images with no noise (red) and with noise of std. deviation of 20% of the dynamic range of the image (blue). After 40 iterations, the PSNR of the proposed method is ~0.6 dB higher than L+S for the noiseless case and more than 1.1 dB higher in the noisy case. Using the mean SSIM measure there are improvements of 0.07 (noiseless case) and 0.04 (noisy case). Fig.2: Top row: L+S reconstruction of the *in vivo* data set. Bottom row: reconstruction using proposed method. Left: temporal frame. Right: cross-section of temporal frames (at red dashed line). The temporal fluctuation artifacts are seen to be greatly reduced in the reconstruction of the proposed method (in most of the image).

Conclusions:

Plugging the proposed snippet-based model into the original L+S scheme leads to quantitative and qualitative improvements. Most importantly, the temporal fluctuation artifacts are much reduced by application of this model. The benefit of this is especially clear when scrolling through the frames. Future work will include application of the snippet-based model in other dynamic MR imaging settings (e.g. cardiac cine), and in combination with other CS schemes and sampling patterns.

References:

[1] Otazo et al, Magn Reson Med, 2014. [2] Aharon et al, IEEE Trans Sig Proc, 54(11):4211-4322, 2006. [3] Chandarana et al., Invest Radiol, 46:648-653, 2011.
[4] http://cai2r.net/resources/software/ls-reconstruction-matlab-code

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