Accelerating Dynamic MRI via Tensor Subspace Learning

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Introduction. Dynamic MRI aims to map out a high temporal and spatial resolution sequence of images. This task is however hindered by low acquisition speed of MR imagers per snapshot that causes a lot of artifacts for imaging of moving objects. Towards accelerating dynamic MRI, the state-of-the-art techniques vectorize image frames (thus compromising spatial correlation) for all time frames and use low rank models to represent dynamic images [1-6]. Our advocated approach builds on three-way tensors and leverages spatiotemporal correlations of the ground truth images through tensor low rank. CP/PARAFAC decomposition of tensors is adapted [7], and a tomographic approach is put forth that leverages the tensor low rank to recursively learn the low-dimensional subspace from undersampled *k*-space data. In the nutshell, the novel approach allows real-time data acquisition without gating or breath-holding, yet being able to recover high-quality dynamic cardiac images from high-dimensional even under-sampled tensors `on-the-fly'. It means the images can be reconstructed while the data is still being acquired.

Methods.

Imaging techniques. To systematically evaluate the performance of the proposed method, free breathing human cardiac MR data was simulated with quasi-periodic heartbeats. Dynamic cardiac cine images of size 200×256 across 256 frames were considered. The *k*-space data was randomly undersampled in all frames with different sampling patterns.

Reconstruction approach. Consider the temporal sequence of the ground-truth images $\{\mathbf{X}_\tau\}_{\tau=1}^{t} \in \mathbb{R}^{M\times N}$, forming a tensor $\underline{\mathbf{X}}_t \in \mathbb{R}^{M\times N\times t}$. PARAFAC decomposition asserts that $\underline{\mathbf{X}}_t$ is spanned by R Rank-one tensors, namely $\underline{\mathbf{X}}_t = \sum_{r=1}^{R} \mathbf{a}_r \circ \mathbf{b}_r \circ \mathbf{c}_r$ where $\circ$ is the outer product [7]. Collecting the bases in the factor matrices $\mathbf{A} := [\mathbf{a}_1, \dots, \mathbf{a}_R] \in \mathbb{R}^{M\times R}$, $\mathbf{B} := [\mathbf{b}_1, \dots, \mathbf{b}_R] \in \mathbb{R}^{N\times R}$ and $\mathbf{C} := [\mathbf{c}_1, \dots, \mathbf{c}_R] \in \mathbb{R}^{t\times R}$, one can express *t*-th tensor slice (image) as $\mathbf{X}_t = \mathbf{A}\,\mathrm{diag}(\boldsymbol{\gamma}_t)\mathbf{B}^T$, where $\boldsymbol{\gamma}_t$ is the *t*-th row of $\mathbf{C}$. Apparently, the factors $(\mathbf{A}, \mathbf{B})$ are common across the slices, and span the tensor row and column space. Spatiotemporal correlation of ground truth images renders the tensor low CP-rank (R <<). The *k*-space data acquired at time *t* however are not perfect, and in general they abide to $y_{i,j}^{(t)} = [\mathcal{F}(\mathbf{X}_t)]_{i,j} + v_{i,j}^{(t)}, (i,j) \in \Omega_t$, where $\mathcal{F}(.)$ is the two-dimensional DFT operator, and $v_{i,j}^{(t)}$ accounts for the noise and unmodeled dynamics. The set $\Omega_t \in \{1, \dots, M\} \times \{1, \dots, N\}$ collects the acquired indices. Given the data $\{y_{i,j}^{(\tau)}, (i,j) \in \Omega_\tau\}_{\tau=1}^{t}$, at time instant *t*, the goal is to learn $(\widehat{\mathbf{A}}_t, \widehat{\mathbf{B}}_t)$, and as a byproduct reconstruct the *t*-th frame $\widehat{\mathbf{X}}_t = \widehat{\mathbf{A}}_t\,\mathrm{diag}(\widehat{\boldsymbol{\gamma}}_t)\widehat{\mathbf{B}}_t^T$ `on the fly'. Towards this end, the factor matrices are first learned via solving the rank-regularized least-squares program

$$(\hat{\mathbf{A}}_t, \hat{\mathbf{B}}_t) = \arg\min_{\mathbf{A},\mathbf{B}} \frac{1}{2t}\sum_{\tau=1}^{t}\sum_{(i,j)\in\Omega_t}\min_{\gamma_t}\left\{\left(y_{i,j}^{(t)} - [\mathcal{F}(\mathbf{A}\mathrm{diag}(\gamma_t)\mathbf{B}^\top)]_{i,j}\right)^2 + \lambda\|\gamma_t\|^2\right\} + \frac{\lambda}{2}\left(\|\mathbf{A}\|_F^2 + \|\mathbf{B}\|_F^2\right).$$

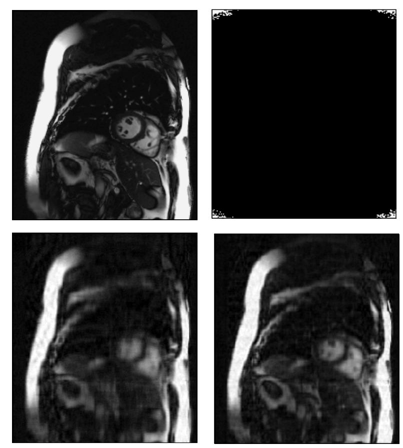

Fig. 1: (top-left) Ground truth, and (top-right) acquired k-space image undersampled by factor 10; reconstructed with 10% data (bottom-left) with R=100, and (bottom-right) *R*=150.

In order to efficiently solve this program, recursive solvers are developed using the first-order stochastic alternating minimization along the lines of our precursor~[8]. Interestingly, iterates are provably convergent, and admit simple parallel updates that mainly involve DFT operations amenable to efficient implementation using the off-the-shelf FFT algorithms. With these neat features the estimator is capable of dealing with very high resolution MRI images using only limited storage and computational resources.

Data analysis. k-space data are sequentially fed into the iterative algorithm, where each iteration updates the subspace matrices $(\mathbf{A}, \mathbf{B})$ with the partial data $y_{i,j}^{(t)}, (i,j) \in \Omega_t$ randomly subsampled at various factors 10 and 4. A few passes over the data are allowed to improve the learning accuracy. The parameters *R*=50, 150 and *λ*=0.1 are tuned according to a small subset of fully acquired frames that can be obtained via a complementary scan. The reconstruction performance is quantified via the average error $\bar{e} := (1/256)\sum_{\tau=1}^{256} e_\tau$, for the instantaneous error $e_t := \|\mathbf{X}_t - \widehat{\mathbf{X}}_t\|_F / \|\mathbf{X}_t\|_F$.

Results. A candidate reconstructed frame using the novel method is compared against the gold standard (full acquisition) in Fig. 1. It is seen that with only 10% available data and for the rank *R*=150 the reconstructed image reveals almost all the details of the gold standard cardiac snapshot. In this case the average error $\bar{e} = 0.06$ is achieved. One can further enhance the image quality by allowing 25% of *k*-space data that leads to the average reconstruction error $\bar{e} = 0.03$ with only four passes over the data. For both cases, higher reconstruction accuracies are possible by choosing smaller step sizes and allowing more passes over the data.

Conclusions. A novel tomographic approach is put forth to accelerate the MRI by leveraging the low intrinsic dimensionality of ground-truth images through the tensor low CP rank. Preliminary tests demonstrate the feasibility of acceleration for dynamic cardiac MRI by factor 4 while maintaining a reasonable accuracy. The novel approach also opens new doors that can potentially lead to significant acceleration. One such idea pertains to utilizing the free real-time estimates $(\widehat{\mathbf{A}}_t, \widehat{\mathbf{B}}_t)$ at *t*-th acquisition snapshot to adaptively select a subset of important *k*-space pixels to acquire in the next snapshot. Another idea is to incorporate the spatial-domain image correlations as a prior knowledge based on a Bayesian interpretation of the sought formulation.

References. [1] Z. P. Liang et al, ISBI: 988-991, 2007. [2] J. P. Haldar et al, ISBI: 716-719, 2010. [3] S. G. Lingala et al, TMI, 30:1042-1054. [4] T. Zhang et al, MRM, 2014. [5] J. Trzasko et al, ISMRM, 2011. [6] R. Otazo et al, MRM, 2014. [7] T. G. Kolda et al, SIAM, 51:455-500. [8] M. Mardani et al TSP, 2014.