# Reconstruction Strategies for Pure 2D Spatiotemporal MRI

Albert Jang[1,2], Alexander Gutierrez[3], Di Xiao[2], Curtis A. Corum[1], Vuk Mandic[4], Jarvis Haupt[2], and Michael Garwood[1]

*[1]Center for Magnetic Resonance Research and Department of Radiology, University of Minnesota, Minneapolis, MN, United States, [2]Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN, United States, [3]Department of Mathematics, University of Minnesota, Minneapolis, MN, United States, [4]School of Physics and Astronomy, Department of Physics, University of Minnesota, Minneapolis, Minneapolis, MN, United States*

**Introduction:** Spatiotemporal-based encoding is a relatively new and emerging methodology, enabling an alternative way of doing MRI. It offers certain advantages over traditional Fourier-based MRI such as robustness to static field ($B_0$) inhomogeneity[1] and the ability to compensate for $B_1$ inhomogeneity[2]. To take full advantage of its spatially-and temporally dependent signal excitation, imaging performance might be improved using image reconstruction strategies that are not common to standard MRI practice. Early work showed a reconstruction scheme that exploits the inherent quadratic phase of a chirp pulse to localize signal[3]. This was followed by substitution of frequency encoding[4] and phase encoding[5] with time encoding. For pure spatiotemporal encoding, one possible solution utilizes algebraic reconstruction, which entails performing a Bloch simulation to obtain the encoding matrix and then taking its pseudo-inverse[2]. In this work, we explore new reconstruction approaches for pure spatiotemporal encoding and compare them with Cartesian gridded Fourier Transform (FT) and pseudo-inverse methods.

**Principle:** We explore two new approaches for reconstructing spatiotemporally encoded images. The first approach is an iterative maximum-likelihood estimation (MLE). We formulate the observed pseudo-FID at time index $i$ as $d_i = \sum_{j=1}^{N} H_{ij} p_j$, where $H_{ij}$ is the encoding matrix describing the response of the system for a scan through a volume of $N$ pixels, and $p_j$ denotes the occupancy of the volume pixel $j$. We define log-likelihood as $\ln L \sim -\sum_{i=1}^{M} \frac{\left(d_i - \sum_j H_{ij}\hat{p}_j\right)^2}{\sigma^2}$, where $M$ is the number of measurements in time and $\hat{p}_j$ is the estimator of the amplitude in the pixel $j$. Assuming measurement error $\sigma$ is independent of time and $\hat{p}_j$ takes any of $P$ discrete values (grey scale resolution of image), for each $j$, we iteratively vary $\hat{p}_j$ among its $P$ possible values and maximize $\ln L$. Iteration stops when improvement in $\ln L$ drops below a predetermined tolerance level. In the second approach, a matrix containing the net magnetization of a uniform object during readout is constructed. The encoding matrix $A$ is given by concatenation of these net magnetizations matrices at each phase offset. Because we rewind the gradients during readout, this is equivalent to the sequential re-phasing of spins. We use this encoding matrix to perform total variation regularization $\min_x\{\|Ax - b\|^2 + \lambda \cdot \mathrm{TV}(x)\}$, where $\mathrm{TV}(x)$ is total variation of the image $x$ and $b$ is the pseudo-FID. To execute minimization, the Monotone Fast Iterative Shrinkage Thresholding (MFISTA) algorithm[6] is employed.

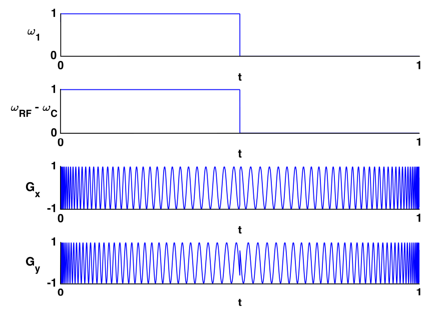
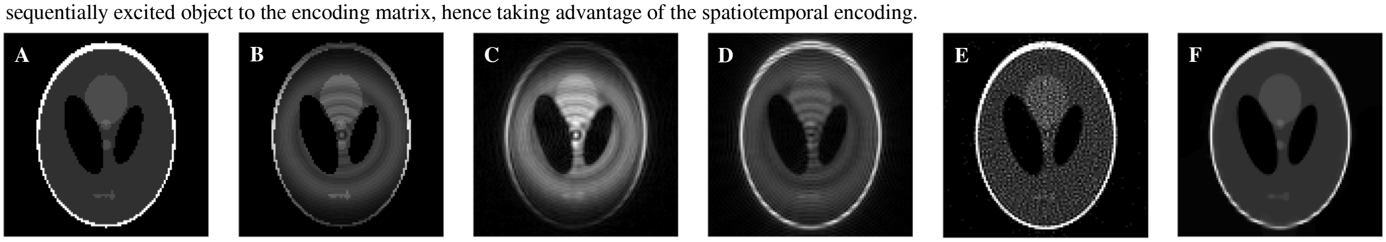



**FIG 1:** Pulse sequence diagram used for Bloch simulations of 10x10 isochromat voxel averaging. Excitation was done using a 2D Chirp pulse ($T_p$ = 10ms, BW = 200kHz, $G_{max}$ = 1.33G/cm) followed by spiral rewind readout for single shot imaging.

**Methods:** The following data were generated with 2D Bloch simulations using 10x10 isochromat voxel averaging on a 100x100 grid. A 2D chirp pulse[7] was used for excitation followed by a spiral rewind during readout as shown in FIG 1. Two sets of simulations, one using a uniform phantom and another using a Shepp Logan phantom, were performed. The uniform phantom data were used to generate encoding matrices. The Shepp Logan data were used to generate images for the following reconstruction schemes: Cartesian gridded FT (convolved with Kaiser-Bessel function), pseudo-inverse (truncated SVD), MLE and total variation regularization (TVR). In the case of MLE, $B_0$ inhomogeneity (-250Hz to 250Hz, linear along the x-direction) was imposed.

**Results and Discussion:** The reconstructed images are shown in FIG 2. The Cartesian gridded FT image (FIG 2C) suffers from resolution when reconstructed onto a 100x100 grid due to data being sampled along a spiral trajectory of 35 turns, which is equivalent to approximately 70 lines in Cartesian k-space. The pseudo-inverse reconstruction (FIG 2D) shows improvement compared to FIG 2C, however still suffers in resolution. Both methods depict the excitation profile that results from a 2D Chirp pulse (FIG 2B). MLE produces an enhancement in terms of resolution as can be ascertained by sharper edges in the image (FIG 2E), representing a clear advantage over the previous methods. Furthermore, MLE is able to compensate for $B_0$ inhomogeneity (data used for reconstruction were generated with a linearly varying $B_0$ field imposed). However, the image is noisy along areas of constant amplitude, which is due to the encoding matrix being under-determined. This can be remedied by increasing the pseudo-FID sampling rate to better condition the matrix. TVR yielded an image (FIG 2F) that was closest to the original Shepp Logan phantom. Reducing the total variation seems to overcome the under-determined limitations of MLE, making it well suited for reconstructing spatiotemporal-encoded signals. A positive outcome of both MLE and TVR is that both are indifferent to the non-uniform excitation profile of the 2D chirp pulse because they rely on linking the sequentially excited object to the encoding matrix, hence taking advantage of the spatiotemporal encoding.




**Shepp Logan** | **Excitation profile** | **Gridded FT** | **pseudo-inverse** | **MLE** | **TVR**

**Fig 2A:** Shepp Logan phantom used in Bloch simulations and **B:** excitation profile resulting from 2D Chirp excitation. Reconstructed image using **C:** Cartesian gridding and taking the FT, **D:** pseudo-inverse, **E:** MLE and **F:** TVL. For MLE, a linearly varying $B_0$ map from -250Hz to 250Hz was used.

**Conclusion:** Various reconstruction schemes were studied and compared for spatiotemporal encoding using simulations. Conventional reconstruction methods such as Cartesian gridded FT and pseudo-inverse appear to perform poorer in terms of resolution, as compared to two new alternative methods. In addition, MLE and TVR approaches, both of which consider the forward model (encoding matrix) of the spatiotemporal-encoding process, yielded images that compensated for the non-uniform excitation profile. In summary, by using MLE or TVS, spatiotemporal encoding may provide an even greater immunity to magnetic field inhomogeneity than was previously possible.

**References:** [1] Ben-Eliezer N, Magn Reson Mater Phy 2012, 25:433-442, [2] Snyder, MRM 2014, 72:49-58, [3] Pipe J, MRM 1996, 36:137-146, [4] Shrot Y, JMR 2005, 172:179-190, [5] Chamberlain R, MRM 2007, 58:794-799, [6] Beck A, IEEE Trans. Image Process 2009, 18:2419-2434, [7] Jang A, ISMRM 2014, Milan, Italy Abstract #2386.

**Acknowledgements:** NIH BTRC - P41 EB015894, R24 MH105998