A Parallel Algorithm for Compressed Sensing Dynamic MRI Reconstruction

Loris Cannelli[1], Paolo Scarponi[1], Gesualdo Scutari[1], and Leslie Ying[1]
[1]*Electrical Engineering, University at Buffalo, Buffalo, NY, United States*

INTRODUCTION:
In compressed sensing MRI [1], iterative algorithms are necessary to reconstruct the desired image from undersampled k-space data. These algorithms are usually computational expensive and this issue is especially serious in reconstructing 3D dynamic images simultaneously. Although a number of algorithms have been developed to improve the convergence rate of the convex ℓ_1 minimization problem [2], the execution time is still too long to be used in clinical routine [3-5]. In this paper, we propose a novel algorithm that significantly increases the convergence rate compared to the existing ones. In addition, we parallelize the algorithm in multiple CPU cores such that the computation time can be shortened by an almost arbitrary high factor, depending on the number of CPU cores available and the size of the problem. Experimental results on cardiac cine imaging show that our proposed algorithm is able to reconstruct high quality dynamic image series within one second, while the existing methods take minutes to achieve similar quality.

THEORY AND METHOD:
In compressed sensing dynamic MRI, the dynamic image series $\boldsymbol{\rho}$ is assumed to be sparse in a spatial and temporal frequency domain. The image series in the *x-f* space is reconstructed by solving an ℓ_1 minimization problem: $\mathbf{min}\{\|\mathbf{F}\boldsymbol{\rho}-\mathbf{d}\|_2^2+\boldsymbol{\lambda}\|\boldsymbol{\rho}\|_1\}$ (1), where $\mathbf{d}$ is the acquired data in *k-t* space, $\mathbf{F}$ is the Fourier transform along the *k-t* direction and $\boldsymbol{\lambda}$ is the regularization parameter which controls the degree of sparsity of the solution. Our algorithmic framework is very flexible and is designed for solving in parallel a wide class of possibly non-convex regularized optimization problems, in which the compressed sensing MRI is comprised. The main idea is to decompose the original problem into a series of smaller, and thus simpler approximated sub-problems, without "destroying" the structure of the main one. The approximating objective functions of each sub-problem must satisfy certain criteria and we refer here to the first version of the algorithm described in [6]. It is a Full-Jacobi type scheme wherein all the cores update in parallel their own block variables by solving a lower dimensional sub-problem. More specifically, the i^{th} core solves at iteration k the following optimization problem: $\hat{\mathbf{x}}_i(\mathbf{x}^k,\boldsymbol{\tau}_i)\triangleq\mathbf{argmin}\left\{\|\mathbf{F}_i\boldsymbol{\rho}_i-\mathbf{d}_i\|_2^2+(\boldsymbol{\tau}_i/\mathbf{2})\left\|\boldsymbol{\rho}_i-\boldsymbol{\rho}_i^k\right\|_2^2+\boldsymbol{\lambda}\|\boldsymbol{\rho}_i\|_1\right\}$ (2). Note that the solution of (2) can be computed in closed form using the (scalar) soft-thresholding operator (see, e.g., [2]). The block $\mathbf{x}^k$ is then updated by taking a convex combination of the optimal solution $\hat{\mathbf{x}}_i(\mathbf{x}^k,\boldsymbol{\tau}_i)$ and the current point $\mathbf{x}^k$, i.e., $\mathbf{x}^{k+1}=\mathbf{x}^k+\boldsymbol{\eta}^k(\hat{\mathbf{x}}^k-\mathbf{x}^k)$ (3)**,** where $\boldsymbol{\eta}^k$ is a suitable step-size (see [6]). It is important to remark that a greedy selection of the block variables to be updated at each iteration can be also performed without affecting the convergence of the scheme; such a procedure has been shown numerically to improve the convergence speed, see [6] for details.

RESULTS AND DISCUSSION:
The data was acquired on a 1.5T Philips scanner. The SSFP sequence was used with a flip angle of 50 degree and TE/TR = 1.7/3.5msec. The fully acquired *k-t* measurements have a size of 256×220×25 (#frequency encoding × #phase encoding × #frame). The FOV was 345mm × 270mm and the slice thickness was 10 mm. The heart rate was 66 bpm. The full *k*-space data were acquired and used as the reference. The variable-density random sampling pattern was generated to simulate a reduction factor of 3, with central 8 fully sampled phase encodings. The proposed FLEXA [6], *k-t* FOCUSS [3] and FISTA [2] methods were used for reconstruction. All the codes have been written in C++ using the Intel Math Kernel Library (MKL) to perform the algebra. All the algorithms except k-t-FOCUSS, for which a sequential version was used, have been implemented in a parallel fashion and all of them were tested on the General Compute Cluster of the Center for Computational Research at SUNY Buffalo. The partition used was composed of 372 DELL 1.6x2.20GHz Intel E5-2660 "Sandy Bridge" Xeon Processor computer nodes with 48 Gb of DDR4 main memory and QDR InfiniBand 40Gb/s network card. For our experiments we used 16 cores (2 nodes with 8 cores each) for parallel algorithm and 1 node (1 core) for the serial ones. Figure 1 and Figure 2 show, respectively, the reconstructions at a single time frame and the intensity profile of a fixed row, both obtained when all methods terminate at 1 second. The convergence curve and frame-by-frame mean squared error (MSE) curve are shown in Fig. 3. We calculated the normalized difference between the adjacent iterations and then terminate iterations once the difference is below a threshold. As can be seen, the proposed FLEXA takes only 1 second to converge to the optimal reconstruction, while *k-t* FOCUSS needs more than 12 seconds to reach similar quality. Moreover, FLEXA captures better the motion with respect to the other algorithms, especially *k-t* FOCUSS, for which the intensity profile is almost constant.

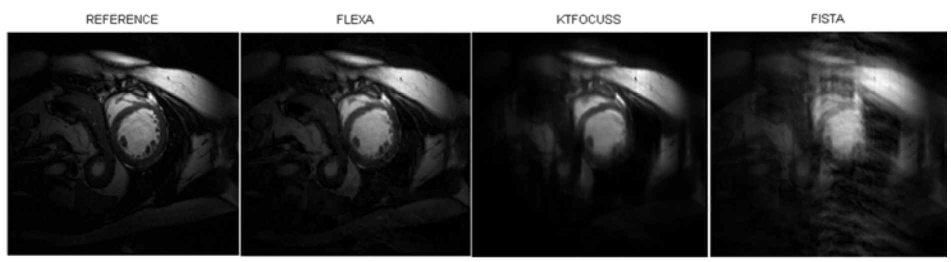

Fig. 1 Last frame reconstructed after 1 second

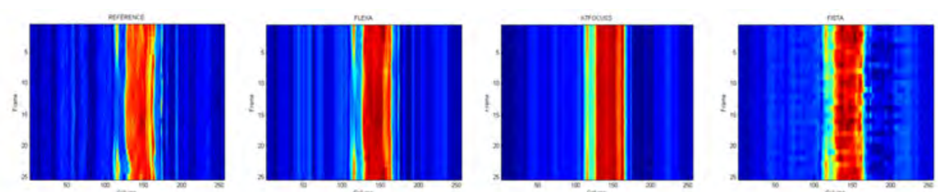

Fig. 2 Intensity profile of the 100th row after 1 second

CONCLUSION:
A novel algorithm is proposed to improve the convergence rate for compressed sensing reconstruction and thus save the computation time. Experimental results demonstrate that the proposed method is able to achieve a lower error in a much shorter time when compared with *k-t* FOCUSS and FISTA and it also reconstruct in a better way the dynamic behavior of the image.

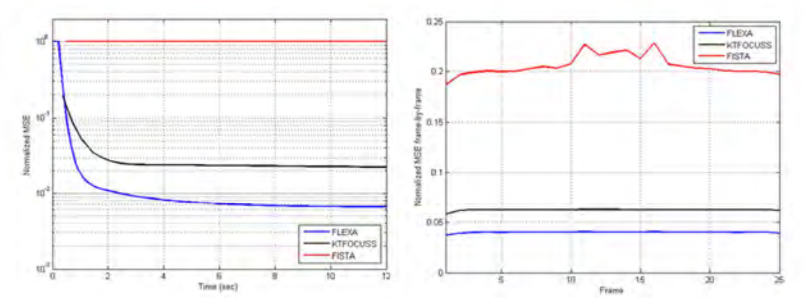

Fig. 3 (Left) convergence curve and (Right) frame-by-frame MSE.

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