Closed-Form Solution Concomitant Field Correction Method for Echo Planar Imaging on Head-only Asymmetric Gradient MRI System

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Target Audience: MRI physicists and hardware engineers.

Background: According to Maxwell's equations, the magnetic field gradient used for spatial encoding in MRI is never exactly linear (as desired) but always includes spatially-varying higher order field components known as concomitant fields[1]. During MRI data acquisition, concomitant fields induce undesired phase accumulation. In applications like echo planar imaging (EPI) that employ high gradient amplitudes[2], this phase accumulation (if unaccounted for) may result in spatially distorted images. Conventional MRI gradient systems have cylindrical symmetric structure, and the concomitant field for such systems contains only 2nd-order space and gradient amplitude dependencies[1]. However, some emerging MRI platforms employ an asymmetric gradient system, such as a compact asymmetric head-only gradient coil[3], which have concomitant fields that also contain zero- and first-order spatial dependencies. The additional first order terms cause further image distortion and echo shift[4]. Previous proposed first-order concomitant field correction methods for EPI acquisitions on asymmetric gradient systems are limited to correcting strictly axial imaging plane setups[4]. In this work, we develop a generalized waveform pre-emphasis framework to correct first-order concomitant fields for arbitrary axial-coronal oblique EPI acquisitions on a head-only asymmetric gradient system, and provide closed-form mathematical expressions for determining first-order gradient pre-emphasis factors for all gradient directions.

Methods: The concomitant fields for an asymmetric MRI gradient system can be expressed as $B_c = [G_x(z+z_{0x}) - G_z\alpha(x+x_0)]^2/(2B_0) + [G_y(z+z_{0y}) - G_z(1-\alpha)(y+y_0)]^2/(2B_0)$, where dimensionless symmetry parameter α describe the relative strength of z gradient-induced concomitant field along the x and y axes, x_0 and y_0 describe the offset of z gradient coil relative to magnet isocenter, and z_{0x} and z_{0y} describe the offset of x and y gradients relative to isocenter[1,4]. The first order concomitant field can be expressed as: $G_x^2 z z_{0x}/B_0 + G_y^2 z z_{0y}/B_0 + (1-\alpha)^2 G_z^2 y y_0/B_0 + \alpha^2 G_z^2 x x_0/B_0 - \alpha G_x G_z(x z_{0x} + z x_0)/B_0 - (1-\alpha) G_y G_z(y z_{0y} + z y_0)/B_0$. Denoting the ideal (actual) gradients as G_x^0, G_y^0, G_z^0, the first order concomitant field can be canceled by setting the nominal gradient field to satisfy the following constraints: $G_z^0 = G_z + G_x^2 z_{0x}/B_0 + G_y^2 z_{0y}/B_0 - \alpha G_x G_z x_0/B_0 - (1-\alpha) G_y G_z y_0/B_0$, $G_x^0 = G_x + \alpha^2 G_z^2 x_0/B_0 - \alpha G_x G_z z_{0x}/B_0$, and $G_y^0 = G_y + (1-\alpha)^2 G_z^2 y_0/B_0 - (1-\alpha) G_y G_z z_{0y}/B_0$, from which a 6-th order equation of G_z can be obtained:

$$G_z^0 = G_z + \frac{z_{0x}}{B_0}\left[\frac{B_0 G_x^0 - \alpha^2 G_z^2 x_0}{B_0 - \alpha G_z z_{0x}}\right]^2 + \frac{z_{0y}}{B_0}\left[\frac{B_0 G_y^0 - (1-\alpha)^2 G_z^2 y_0}{B_0 - (1-\alpha) G_z z_{0y}}\right]^2 - \frac{\alpha G_z x_0}{B_0}\frac{B_0 G_x^0 - \alpha^2 G_z^2 x_0}{B_0 - \alpha G_z z_{0x}} - \frac{(1-\alpha) G_z y_0}{B_0}\frac{B_0 G_y^0 - (1-\alpha)^2 G_z^2 y_0}{B_0 - (1-\alpha) G_z z_{0y}} \quad [Eq. 1].$$

For a conventional symmetric gradient system, $\alpha = 0.5$ and $x_0 = y_0 = z_{0x} = z_{0y} = 0$, and thus the first-order terms vanish. However, for the head-only asymmetric gradient system of interest here, $\alpha = 0.5$, $x_0 = y_0 = 0$, $z_{0x} = z_{0y} = z_0 = 12.0$ cm, and the first-order terms remain. Consequently, Eq. 1 becomes a cubic equation with real-valued coefficients: $(z_0)^2(G_z)^3/4 + [-G_z^0(z_0)^2/4 - B_0 z_0](G_z)^2 + [(B_0)^2 + B_0 z_0 G_z^0]G_z - (B_0)^2 G_z^0 + B_0 z_0\left[(G_x^0)^2 + (G_y^0)^2\right] = 0$, which has at least one real root with a closed-form solution. The expression of G_z can then be used to solve G_x and G_y as: $G_x = (B_0 G_x^0)/(B_0 - G_z z_0/2)$, and $G_y = (B_0 G_y^0)/(B_0 - G_z z_0/2)$. Hence, all the gradient waveforms with pre-emphasis can be obtained. To test the proposed method, the gradient waveforms of an axial/coronal plane (oblique angle = 20°) EPI acquisition was plotted (read out gradient amplitude = 23 mT/m, slew rate = 149 T/m/s). The pre-emphasis components for each gradient axis, and the post-correction residual first-order fields were then calculated.

Results: Fig. 1a shows the ideal x, y, z gradient waveforms (G_x^0, G_y^0, G_z^0), and Fig. 1b shows the gradient bias caused by the first-order concomitant field. Fig. 1c shows the post-correction residual gradient bias after the nominal gradients has been pre-emphasized using the proposed approach.

Discussion: As shown in Fig. 1, the first-order concomitant field can be completely eliminated by pre-emphasizing gradient waveforms using the proposed method. With all first-order terms eliminated, the zero-order term can then be removed by subtracting accumulated phase, and second order terms can be eliminated using established image-based corrections[1,2,5]. In the gradient amplitude configuration used in this work (23 mT/m), the first- order concomitant field causes a 100 Hz frequency shift at 12 cm from isocenter. The frequency offset is expected to be much stronger when the full gradient capabilities (80 mT/m) of our gradient system are used, due to the quadratic relation between gradient amplitude and concomitant field strength[3]. To simplify waveform design, the quadratic ramp waveforms in Fig. 1b could also be approximated by a linear ramp, in which case the area of pre-emphasis waveform should be kept the same so that phase accumulation at each echo center is nulled[2]. Eq. 1 can be simplified to a cubic equation due to the zero offset of our z gradient system. In the more general, but rarely encountered, case where all x, y, z gradient offsets are non-zero, the root of Eq. 1 could be solved numerically (e.g., via Newton's method) under auxiliary real constraints.

Conclusions: A generalized framework is developed to correct first order concomitant field for arbitrary axial-coronal oblique EPI acquisition planes on a head-only asymmetric gradient system.

Acknowledgment: This work was supported in part by the NIH grant 5R01EB010065.

References: [1] Bernstein MA et al., MRM 39:300-8, 1998; [2] Zhou XJ et al., MRM 39:596-605, 1998; [3] Lee S, et al. ISMRM 22 (2014), 0310; [4] Meier C et al., MRM 60:128-34, 2008; [5] Du Y et al., MRI 48:509-15, 2002.

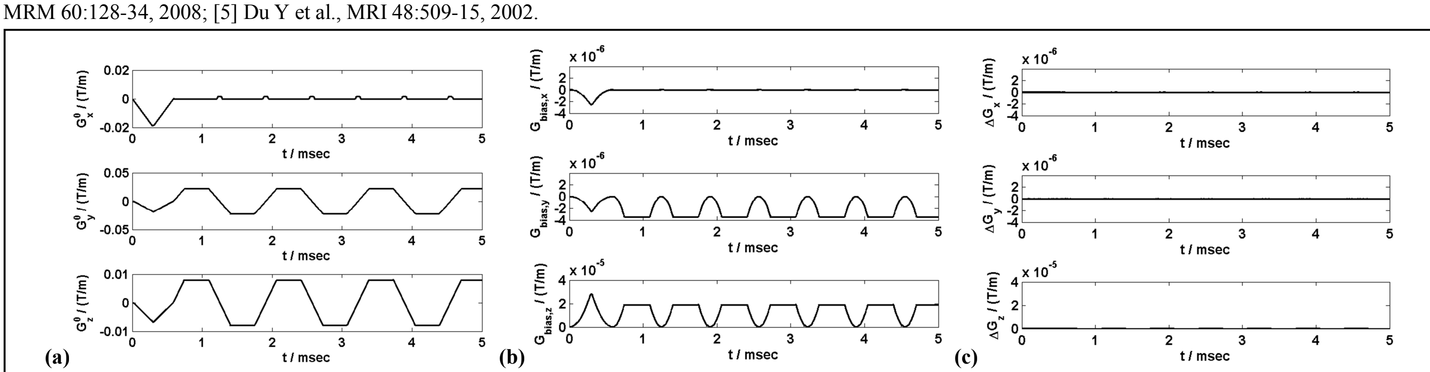

Figure 1. (a) Ideal gradient waveforms in x, y, z axis for axial/coronal plane EPI; (b) gradient bias caused by first order concomitant field; (c) residual bias gradient after correction