Phantom-Based Iterative Estimation of MRI Gradient Nonlinearity

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Target Audience: MRI hardware engineers, medical physicists, and image reconstruction scientists interested in quality control and assurance.

Purpose: Due to various engineering and safety limitations, generating truly linear magnetic field variations (i.e., gradients) over an imaging field-of-view (FOV) to spatially encode an MRI signal is infeasible in practice [1]. To construct geometrically accurate MR images, gradient nonlinearities (GNL) – i.e., deviations from linear ideality – must be accounted for during [2] or after (i.e., post-processing) [3] image reconstruction from the raw k-space data. To operate effectively, these methods require accurate information about the actual gradient fields used during data acquisition. Typically, field models are generated via platform-specific electromagnetic (EM) simulation. In addition to relying on proprietary information, such models do not capture scanner-specific hardware construction (e.g., coil winding) or siting variations [4]. In [5], Tao et al. proposed a gradient calibration procedure where: 1) a phantom containing fiducial markers whose physical positions are *a priori* known [6] is scanned; 2) a software program identifies marker positions within the (uncorrected) images; and 3) an estimate of the field is constructed using the differences between nominal and actual marker positions. Images corrected using these scanner-specific gradient models generated were shown to be geometrically more accurate than those corrected using vendor-provided generic models. In this work, we present a novel gradient field estimation strategy that explicitly minimizes corrected image marker position mean square error (MSE), which is a standard metric of quantitative image accuracy. After showing that the procedure in [5] coincides with one iteration of our algorithm, we demonstrate that performing GNL correction using iteratively estimated gradient fields provides further improvements in geometric accuracy.

Methods: GNL correction can be abstractly defined as $\boldsymbol{x}_{corrected} = F\{\boldsymbol{x},\boldsymbol{C}\}$, where $\boldsymbol{x}$ is the nominal 3D MR image volume, $\boldsymbol{C}$ is an $N \times 3$ coefficient matrix of an N^{th}-order spherical harmonic approximation of the gradient field [3,5], $F\{\cdot,\cdot\}$ is the correction function (e.g., [2,3]), and $\boldsymbol{x}_{corrected}$ is the geometrically corrected image volume. As in [5], presume that $\boldsymbol{x}$ is an image of a phantom that contains M fiducial markers whose true physical position, $\boldsymbol{P_0}$ ($M \times 3$ matrix), is *a priori* known (e.g., the Alzheimer's Disease Neuroimaging Initiative (ADNI) phantom [6]), and $A\{\cdot\}$ is an operator (e.g., the AQUAL software package [6]) that produces image-based estimates of marker position. The spatial distortion of an image due to GNL is summarized by the mean square error (MSE) of measured versus actual marker positions, i.e., $\text{MSE}(\boldsymbol{x}) = \|A\{\boldsymbol{x}\} - \boldsymbol{P_0}\|_F^2$, where $\|\cdot\|_F$ denotes the Frobenius norm. Similarly, the spatial distortion MSE of a GNL corrected image is $\text{MSE}(\boldsymbol{x},\boldsymbol{C}) = \|A\{F\{\boldsymbol{x},\boldsymbol{C}\}\} - \boldsymbol{P_0}\|_F^2$. Presuming that the distorted image, $\boldsymbol{x}$, is fixed, the gradient field can be estimated by finding the spherical harmonic expansion coefficients, $\boldsymbol{C}$, that minimizes spatial distortion MSE, i.e., solving $[\widehat{\boldsymbol{C}}] = \arg\min_{\boldsymbol{C}} \text{MSE}(\cdot,\boldsymbol{C})$. We approach this optimization problem using Gauss-Newton (GN) iteration. Define the residual function $H\{\boldsymbol{C}\} = A\{F\{\boldsymbol{x},\boldsymbol{C}\}\} - \boldsymbol{P_0}$. Given a current solution estimate, $\boldsymbol{C_t}$, GN iteration uses a linear approximation of the $H\{\cdot\}$ to construct a new estimate, $\boldsymbol{C_{t+1}} = \boldsymbol{C_t} + \boldsymbol{\Delta C_{t+1}}$, where $[\boldsymbol{\Delta C_{t+1}}] = \arg\min_{\boldsymbol{\Delta C}} \|H\{\boldsymbol{C_t}\} + J_H\{\boldsymbol{C_t}\} \odot \boldsymbol{\Delta C}\|_F^2$, $J_H\{\boldsymbol{C_t}\}$ is the 4-way Jacobian tensor of $H\{\cdot\}$, and $\odot$ is the multilinear product. Since $A\{\cdot\}$ generally does not have a closed-form mathematical expression, its Jacobian cannot be determined exactly. Thus, we approximate the effect of its application as $J_H\{\boldsymbol{C_t}\} \odot \boldsymbol{\Delta C} \approx -\boldsymbol{S_0}\boldsymbol{\Delta C}$, where the $M \times N$ matrix, $\boldsymbol{S_0}$, represents the spherical harmonic basis evaluated at true marker spatial positions, $\boldsymbol{P_0}$. In words, this approximation presumes that GNL correction is exact and that the difference between marker positions in the corrected and uncorrected images is essentially a function of only the distortion field. This yields the following simplified update rule: $[\boldsymbol{\Delta C_{t+1}}] = \arg\min_{\boldsymbol{\Delta C}} \|H\{\boldsymbol{C_t}\} - \boldsymbol{S_0}\boldsymbol{\Delta C}\|_F^2 = (\boldsymbol{S_0^*}\boldsymbol{S_0})^{-1}\boldsymbol{S_0^*}H\{\boldsymbol{C_t}\}$, which require only basic matrix multiplication. Note that the procedure in [5] coincides with one iteration of this GN sequence given an initial estimate $\boldsymbol{C_0} = \boldsymbol{0}$. To test the efficacy of our proposed iterative gradient field estimation strategy, the ADNI phantom was imaged on a General Electric 3.0 T Signa HDxt (v16.0) system with a 3D MP-RAGE sequence (orientation=axial, Nx=Ny=256, Nz=196, Δx=1.05 mm, Δz=1.3 mm) and high-performance whole-body gradients. The image volume was first reconstructed without gradient nonlinearity correction, yielding geometrically distorted results (see first column of Fig. 1). GNL correction via image-domain cubic spline interpolation [3] was then performed using gradient field coefficients: 1) provided by the vendor [5]; 2) obtained via the non-iterative procedure in [5]; and 3) generated after 25 iterations of the proposed MSE minimization algorithm, initialized with $\boldsymbol{C_0} = \boldsymbol{0}$. Post-correction marker position root MSE (RMSE) was estimated using AQUAL [6]. Our current Matlab implementation of the MSE minimization algorithm requires ~2 min/iteration on a dual 8-core (2.6 GHz) machine with 128 Gb memory.

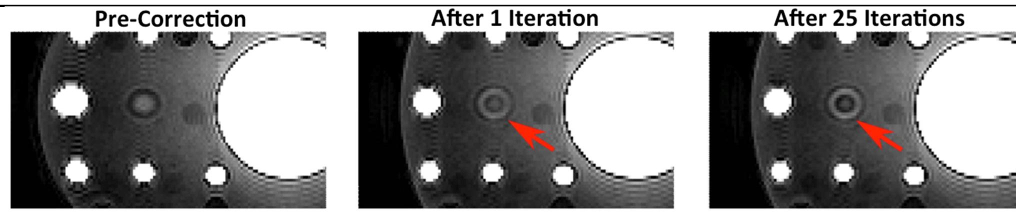

Fig. 1) Phantom GNL correction using gradient field coefficient generated before and after 1 and 25 iterations. Arrows highlight where GNL correction with iteratively estimated field coefficients improves performance.

Results: Fig. 1 shows image results before and after GNL correction, using coefficient sets generated via the proposed algorithm – note that the coefficient set obtained after 1 iteration coincides with that from the non-iterative procedure from [5]. Before GNL correction, fiducial marker RMSE was 3.197 mm. Post-correction RMSE using vendor-provided gradient field coefficients and those generated with the approach in [5] were 0.327 mm and 0.326 mm, respectively. As shown in Fig. 2, subsequent iteration progressively (and monotonically) decreases RMSE to 0.276 mm, which is about 15% lower (better) than the non-iterative result. All corrections meet the 0.35 mm RMSE minimum guideline outlined in [6] for quantitative image analysis.

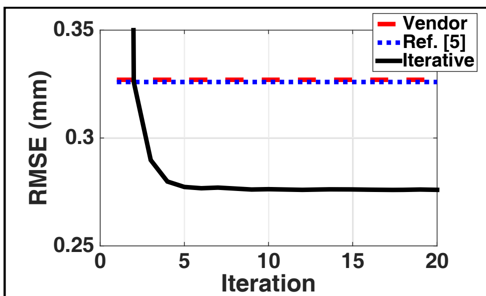

Fig. 2) Fiducial marker root-mean-square-error (RMSE) versus iteration number. Note the rapid, monotonic convergence of the GN algorithm.

Discussion: Like the phantom-based gradient field estimation strategy in [5], our proposed iterative approach does not require any direct magnetic field measurements or proprietary knowledge of gradient coil design, which facilitates straightforward, scanner-specific calibration in a wide range of clinical settings. As demonstrated, this computational strategy – whose explicit objective is to minimize GNL-corrected image marker position MSE – enables improved post GNL-correction geometric accuracy when compared against GNL-corrections performed with vendor-provided or non-iteratively estimated gradient fields.

Conclusion: Iterative gradient field estimation enables improved gradient nonlinearity correction performance and thus quantitative image accuracy.

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References: [1] L. Schad et al., MRI 10:609-21, 1992; [2] S. Tao et al., MRM DOI:10.1002/mrm.25487; [3] G. Glover et al., U.S. Patent 4591789, 1986; [4] A. Janke et al., MRM 52:115-22, 2004; [5] S. Tao et al. Proc. ISMRM 2013, p.4863; [6] J. Gunter et al. Med Phys 2009;36:2193-2205.