Partial Fourier Homodyne Reconstruction with Non-iterative, Integrated Gradient Nonlinearity Correction

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Target Audience: Scientists and engineers interested in advanced reconstruction methods

Background: Partial Fourier acquisition and reconstruction is a widely used method to reduce the amount of data required to form an image in MRI by up to 50% (1,2). Recently, a non-iterative MR image reconstruction method with integrated gradient nonlinearity (GNL) correction was proposed (3). The method was shown to be able to mitigate the image blurring and resolution loss introduced by conventional, image-domain interpolation based GNL correction, while still correcting the coarse-scale image geometrical distortion caused by GNL (3,4). In this work, we discuss the addition of partial Fourier acquisition to this integrated reconstruction paradigm to allow for similar maintenance of spatial resolution while reducing acquisition time.

Methods: In the presence of gradient nonlinearity, the Fourier domain MR signal measurement vector, **g,** can be modeled as in the following affine algebraic form: $\mathbf{g} = \mathbf{A}diag(\boldsymbol{\varphi})\mathbf{f} + \mathbf{n}$, where **f** is the real-valued vector representing the image object of interest; $diag(\boldsymbol{\varphi})$ is a diagonal matrix with $\boldsymbol{\varphi}$ denoting the spatial map of phase accumulation at each pixel (due to B_0 inhomogeneity, off resonance excitation, etc.); **A** denotes the spatial encoding operator explicitly accounting for GNL induced geometrical distortion. For Cartesian MRI, the forward operator **A** represents a mapping from non-uniform *image* space grids (due to GNL) to a uniform *k*-space grid, which can be efficiently implemented via a type I non-uniform fast Fourier transform (NUFFT) operator. Note that without GNL, **A** reduces to a discrete Fourier transform (DFT) operator. The image vector **f** can be obtained by solving the following least squares estimation problem: $\text{argmin}_{\mathbf{f}\in\mathbb{R}^N} \|\mathbf{g} - \mathbf{A}diag(\boldsymbol{\varphi})\mathbf{f}\|_2^2$, which has the closed-form solution as: $\mathbf{f} = Re\{diag(\boldsymbol{\varphi}^*)(\mathbf{A}^*\mathbf{A})^{-1}\mathbf{A}^*\mathbf{g}\}$, where $\mathbf{A}^*$ is the adjoint operator of **A**. Ref. (2) shows that the $(\mathbf{A}^*\mathbf{A})^{-1}$ term can be approximated by the Jacobian determinant (diagonal matrix), **J**, of distortion mapping caused by GNL, leading to $\mathbf{f} \approx Re\{diag(\boldsymbol{\varphi}^*)\mathbf{J}\mathbf{A}^*\mathbf{g}\}$. The signal measurement vector **s** can then be symmetrically split up along phase encoding or readout direction as $\mathbf{f} \approx Re\{diag(\boldsymbol{\varphi}^*)\mathbf{J}\mathbf{A}^*[\boldsymbol{\Phi}_L + \boldsymbol{\Phi}_{H1} + \boldsymbol{\Phi}_{H2}]\mathbf{g}\}$, where $\boldsymbol{\Phi}_L$ is a binary matrix extracting the low-pass region of *k*-space (along one direction), while $\boldsymbol{\Phi}_{H1}, \boldsymbol{\Phi}_{H2}$ symmetrically extract the high-pass regions below or above the central low-pass region. Assuming conjugate symmetry, the image vector **f** can be estimated via homodyne-type reconstruction (1) via:

$$\mathbf{f} \approx Re\{diag(\boldsymbol{\varphi}^*)\mathbf{J}\mathbf{A}^*[\boldsymbol{\Phi}_L + 2\boldsymbol{\Phi}_{H1}]\mathbf{g}\}. \quad Eq\ [1]$$

To test the proposed strategy, the American College of Radiology (ACR) phantom were scanned with a 2D T1 weighted spin echo sequence (TR = 500 ms, TE = 13 ms, matrix size = 256 × 256, axial acquisition plane, BW = ±15.63 kHz, slice thickness = 3 mm. The fully sampled *k*-space data was retained, and then retrospectively undersampled to 61% of its original size in the phase encoding direction. The phase map was estimated using the low-spatial frequency, central *k*-space data (i.e., $\boldsymbol{\Phi}_L\mathbf{g}$) by $\boldsymbol{\varphi} \approx \exp\{\text{Arg}(\mathbf{J}\mathbf{A}^*\boldsymbol{\Phi}_L\mathbf{g})i\}$, where function $\text{Arg}(\cdot)$ extracts the phase of a complex valued vector. Six reconstruction experiments were performed. First, the standard homodyne reconstruction was applied to undersampled *k*-space data without GNL correction, i.e., with $\mathbf{A}^*$ in Eq [2] denoting a regular inverse discrete Fourier transform (iDFT). Then, the standard GNL correction method based on image-domain interpolation was applied to the standard homodyne reconstruction (4). Finally, the same *k*-space data was directly reconstructed using the proposed method with $\mathbf{A}^*$ representing an adjoint type I NUFFT operator. For comparison, the same procedures were repeated for fully sampled *k*-space data.

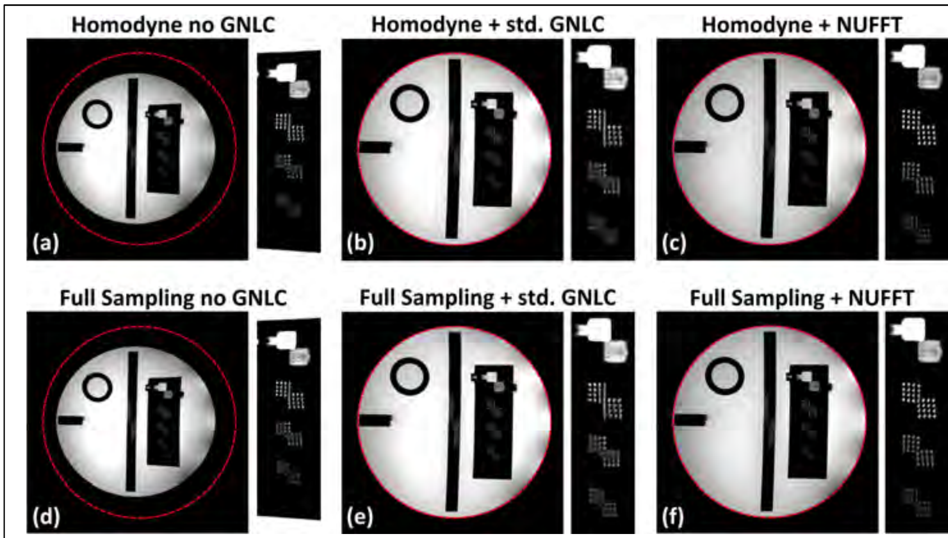

Figure 1. (a) Standard homodyne reconstruction without gradient nonlinearity correction (GNLC); (b) applying standard GNLC (std. GNLC) to (a); (c) the proposed NUFFT based homodyne reconstruction; (d) direct reconstruction on fully sampled *k*-space data without GNLC; (e) applying standard GNLC (std. GNLC) to (d); (f) applying NUFFT to fully sampled *k*-space data.

Results: Figs. 1a to 1c show the images reconstructed from undersampled *k*-space data by: (a) standard homodyne reconstruction without GNL correction (GNLC); (b) standard homodyne reconstruction followed by standard GNL correction (std. GNLC); and (c) the proposed NUFFT based homodyne-type reconstruction. Figs. 1d to 1e show the images reconstructed from fully sampled *k*-space data by (d) direct reconstruction with iDFT without GNL correction; (e) applying standard GNL correction to (d); (f) applying NUFFT to fully sampled *k*-space data.

Discussion: Comparison between the first and second row of Fig. 1 shows that the proposed NUFFT-based homodyne-type reconstruction is able to reduce the blurring effect and resolution loss introduced by conventional GNL correction method, which is consistent with the observation in reconstruction results from fully sampled *k*-space data.

Conclusions: Explicitly accounting for gradient nonlinearity in the homodyne reconstruction allows for maintenance of spatial resolution while reducing scan time.

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