A Fast Reconstruction Algorithm for Accelerated Multi-Contrast MRI

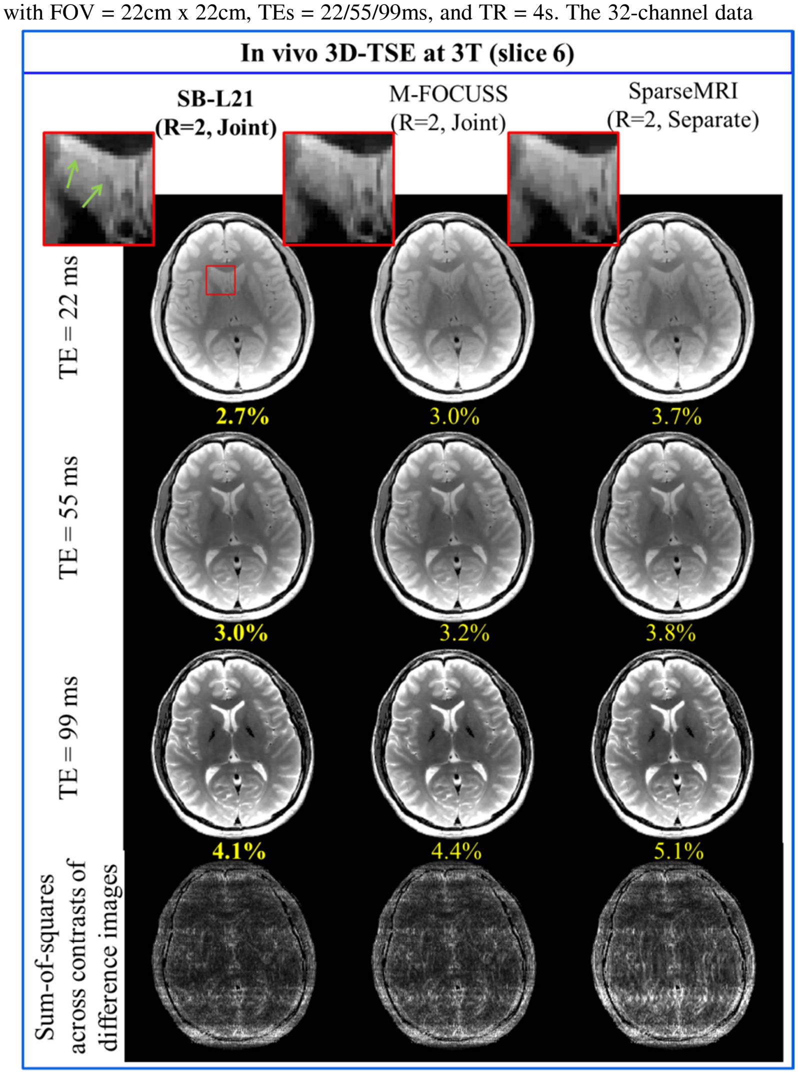
Itthi Chatnuntawech[1], Berkin Bilgic[2], Adrian Martin[1,3], Kawin Setsompop[2,4], and Elfar Adalsteinsson[1,5]

[1]*MIT, Cambridge, MA, United States,* [2]*A. A. Martinos Center for Biomedical Imaging, MA, United States,* [3]*Universidad Rey Juan Carlos, Mostoles, Madrid, Spain,* [4]*Harvard Medical School, MA, United States,* [5]*Harvard-MIT Heath Sciences and Technology, MA, United States*

TARGET AUDIENCE: Researchers interested in fast image reconstruction and multi-contrast imaging.

PURPOSE: In applications where data from the same region of interest are acquired multiple times with different contrast settings and undersampling in k-space, it has been demonstrated that the use of shared features results in higher reconstruction quality[1,2,3]. Here, we present an efficient algorithm to jointly reconstruct a set of images with different contrasts that has faster reconstruction time and better image quality as measured by the root-mean-square error (RMSE). To efficiently solve the $\ell_{2,1}$-regularized optimization problem, our proposed algorithm first adopts the Split-Bregman[4] (SB) technique to break down the problem into sub-problems. We efficiently compute a closed-form solution to each of the sub-problems with the help of a finite difference operator in k-space[5]. The proposed algorithm (SB-L21) offers up to 32x faster reconstruction with up to 30% reduction in an average RMSE of the reconstructed images across all contrasts and slices, compared to other methods, including M-FOCUSS[6] and SparseMRI[7].

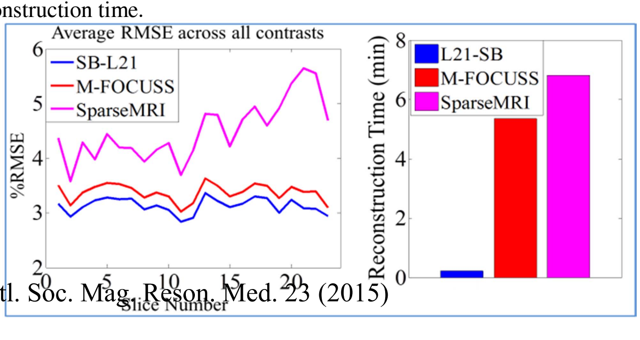
METHODS: In this work, the following $\ell_{2,1}$-regularized optimization problem is solved to reconstruct the data of size $N_x \times N_y$ pixels with T different contrasts: $\min_{\boldsymbol{X}} \frac{1}{4}\sum_{t=1}^{T}\left\|\boldsymbol{M}_t\boldsymbol{F}\boldsymbol{x}_t - \boldsymbol{y}_t\right\|_2^2 + \lambda\left\|\boldsymbol{G}\boldsymbol{X}\right\|_{2,1}$ (Eq. 1) where $\boldsymbol{X} = [\boldsymbol{x}_1, \dots, \boldsymbol{x}_T] \in \mathbb{C}^{(N_x \times N_y)\times T}$; $\boldsymbol{x}_t$ is the vectorized image with a specific contrast; $\boldsymbol{F}$ is a fully sampled 2DFT; $\boldsymbol{M}_t$ is the undersampling mask for the t^{th} image contrast; $\boldsymbol{y}_t$is the observed k-space data on a Cartesian grid; $\boldsymbol{G} = [\boldsymbol{G}_x; \boldsymbol{G}_y]$is a finite difference operator; λ is a regularization parameter. Extending SB formulation, we iteratively solve the following problem: $\min_{\boldsymbol{X},\boldsymbol{Z},\boldsymbol{S}} \frac{1}{4}\sum_{t=1}^{T}\left\|\boldsymbol{M}_t\boldsymbol{F}\boldsymbol{x}_t - \boldsymbol{y}_t\right\|_2^2 + \lambda\left\|\boldsymbol{Z}\right\|_{2,1} + \frac{\mu}{4}\left\|\boldsymbol{Z} - \boldsymbol{G}\boldsymbol{X} - \boldsymbol{S}\right\|_F$. For each iteration, we update $\boldsymbol{X}$ and $\boldsymbol{Z}$ by sequentially solving two sub-problems: (i) $\min_{\boldsymbol{Z}} \frac{\mu}{4}\left\|\boldsymbol{Z} - \boldsymbol{G}\boldsymbol{X} - \boldsymbol{S}\right\|_F + \lambda\left\|\boldsymbol{Z}\right\|_{2,1}$ and (ii) $\min_{\boldsymbol{X}} \sum_{t=1}^{T}\left\|\boldsymbol{M}_t\boldsymbol{F}\boldsymbol{x}_t - \boldsymbol{y}_t\right\|_2^2 + \mu\left\|\boldsymbol{Z} - \boldsymbol{G}\boldsymbol{X} - \boldsymbol{S}\right\|_F$. For (i), $\boldsymbol{Z}$ is updated using a soft-thresholding operator: $\boldsymbol{Z}^{(n)} = (\boldsymbol{G}\boldsymbol{x} + \boldsymbol{S})^{(n)} \circ \max(1 - 2\lambda/\left(\mu\left\|(\boldsymbol{G}\boldsymbol{X} + \boldsymbol{S})^{(n)}\right\|_2\right), 0)$ where $\boldsymbol{Z}^{(n)}$ is a vector of the n^{th} pixel values across multiple contrasts. For (ii), by rewriting the Frobenius norm as a summation of ℓ_2 vector norms, we can solve for each image contrast separately. Furthermore, the finite difference operator is implemented in k-space to arrive at the following closed-form solution: $\boldsymbol{x}_t = \boldsymbol{F}^{-1}[(\boldsymbol{M}^H\boldsymbol{M} + \mu\boldsymbol{E}^H\boldsymbol{E})^{-1}(\boldsymbol{M}^H\boldsymbol{y}_t + \mu\boldsymbol{E}^H\boldsymbol{F}(\boldsymbol{Z} - \boldsymbol{S}))]$ where $\boldsymbol{E} = [\boldsymbol{E}_x; \boldsymbol{E}_y]$. Here, we express the finite difference operator along the x-axis as $\boldsymbol{G}_x = \boldsymbol{F}^{-1}\boldsymbol{E}_x\boldsymbol{F}$ where $\boldsymbol{E}_x$ is a diagonal matrix with 1 - $\exp(i2\pi k_x(m,m)/N_x)$ as its m^{th} diagonal entry. $\boldsymbol{E}_y$is defined similarly. Finally, we update $\boldsymbol{S}$ as follows: $\boldsymbol{S} = \boldsymbol{S} - (\boldsymbol{Z} - \boldsymbol{G}\boldsymbol{x})$. **Experiment:** In this study, we compared the performance of the proposed method to the M-FOCUSS joint reconstruction algorithm and total variation regularized CS (SparseMRI). Dataset with 3 different contrasts and 23 slices were acquired fully sampled from a healthy volunteer at 3T using turbo spin-echo at 0.9x0.9x3 mm^3 resolution with FOV = 22cm x 22cm, TEs = 22/55/99ms, and TR = 4s. The 32-channel data were combined and reconstructed, and the corresponding the k-space data were then retrospectively, randomly undersampled along the phase encoding direction with an acceleration factor of two (R = 2) in MATLAB. Different undersampling patterns were used for different contrasts. For SB-L21 and SparseMRI, we selected a regularization parameter that yielded the smallest RMSE with respect to the fully sampled data: $e = 100 \times \left\|\boldsymbol{x} - \boldsymbol{x}_{true}\right\|_2 / \left\|\boldsymbol{x}_{true}\right\|_2$. Our stopping criterion was that the change in RMSE between consecutive iterations is less than 1%, i.e. when $100 \times \left\|e^{(k)} - e^{(k-1)}\right\|_2 / \left\|e^{(k-1)}\right\|_2 < 1$ where $e^{(k)}$ is RMSE at iteration k.



RESULTS: **SB-L21 took 12.9s** to reconstruct the multi-slice data compared to **5.4 min for M-FOCUSS** and **6.8 min for SparseMRI**. The average RMSE across all contrasts and slices was 3.1% for SB-L21, 3.4% for M-FOCUSS, and 4.5% for SparseMRI. The reconstructed images from SB-L21 have lower RMSEs (shown under each image in yellow) and better visual quality because SB-L21 preserves the edges (green arrows in inset figures). Inset figures (upper left corners of top-row images) are zoomed-in views of the red box.

DISCUSSION: Fast reconstruction of SB-L21 is achieved by formulating Eq. 1 as two sub-problems that are amenable to a closed-form solution. Critical to this formulation is the calculation of finite differences in k-space. Compared to SparseMRI, SB-L21 and M-FOCUSS improve reconstruction quality because they make use of shared features among the contrasts. Better reconstruction quality is also achieved by using different undersampling pattern for different contrast (result not shown) because of reduced coherence of undersampling artifacts across contrasts.

CONCLUSION: As demonstrated through in vivo results, SB-L21 takes only 12.9s to reconstruct the 23-slice, 3-constrast data. It offers 25x faster reconstruction with 7% reduction in RMSE and 32x faster reconstruction with 30% reduction in RMSE with respect to M-FOCUSS and SparseMRI, respectively. The proposed algorithm can also be a rapid alternative to the previously proposed joint Bayesian CS[1], which has a very long reconstruction time.



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