Parallel Imaging Acceleration beyond Coil Limitation using a k-space Variant Low-rank Constraint on Correlation Matrix

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Target Audience: Researchers in the field of parallel imaging and image reconstruction.

Purpose: Recent works [1, 2] have demonstrated that images can be reconstructed by imposing a low-rank constraint to a Hankel-like data matrix formed from multi-channel undersampled data. The presented work explores whether this low-rank constraint, if integrated into the linear equations for parallel imaging, can provide an approach to accelerating MRI data collection beyond the limitation posed by a coil array. Here k-space based parallel imaging is described by a low-rank Toeplitz-like correlation matrix formed from auto- and cross-channel correlation functions. Since the rank of this correlation matrix varies in k-space, missing data can be reconstructed with a k-space variant low-rank constraint in a region-by-region fashion. Imaging acceleration can be improved accordingly using an undersampling factor in those regions with a more stringent constraint. Our purpose is to demonstrate that a k-space variant low-rank constraint on the correlation matrix in parallel imaging permits the use of a net acceleration factor higher than the number of coil elements in the phase-encoding direction.

Methods: By definition, a correlation function between two arbitrary channels i and j is given by:

$$c_{ij}(k) = sum\{\ [d_i(k'+k)]\ conjugate[d_j(k')]\ \}_{\text{over } k'} \qquad (1),$$

where $\{d_i(k),\ i=1,2,\ldots,N\}$ represents N-channel fully sampled k-space data. As previously demonstrated [3], parallel imaging can be formulated as a set of linear equations of which the coefficients are correlation functions. By organizing these coefficients into a correlation matrix $\mathbf{\Gamma_{corr}}$ with all its entries calculated by Eq. 1, parallel imaging reconstruction (PR) from multi-channel undersampled data can be modelled as a function of the correlation matrix, i.e., $PR(\mathbf{\Gamma_{corr}}) = \mathbf{U_{data}}\ (\mathbf{\Gamma_{corr}} \cdot \mathbf{\Pi_{traj}})^{-1}\ (\mathbf{\Gamma_{corr}} \cdot \mathbf{\Lambda_{target}})$, where $\mathbf{U_{data}}$ is the data matrix formed from multi-channel undersampled data, $\cdot$ represents matrix element-by-element multiplication, $\mathbf{\Pi_{traj}}$ is a Boolean matrix that arises from undersampling and $\mathbf{\Lambda_{target}}$ is a Boolean matrix for selecting the target channels to be reconstructed.

Figure 1(a) illustrates the block-Toeplitz structure of a correlation matrix. Mathematically, this matrix is equal to the SAKE [1] calibration matrix left multiplied by its conjugate transpose and thus has the same low-rank nature. In the presented work, it is found that the rank of a correlation matrix calculated from outer k-space data is lower than that from center k-space data and parallel imaging can benefit from a k-space variant low-rank constraint. Herein, we introduce a region-by-region image reconstruction approach. The constrained parallel imaging for each k-space region indexed by k can be formulated as:

$$\text{Minimize } \| P\{PR(\mathbf{\Gamma_{corr}}) \text{ in region } k\} - y \|^2 \ \text{ subject to } rank(\mathbf{\Gamma_{corr}})=r(k) \qquad (2),$$

where P is sampling projection, y represents collected data and $r(k)$ represents priori information about the rank of $\mathbf{\Gamma_{corr}}$ in different k-space regions. To further improve imaging acceleration, a variable density sampling trajectory (Figure 1b) is used. This trajectory has a higher undersampling factor in those k-space regions (outer k-space) with a more stringent low-rank constraint. Figure 1(c) gives the flow chart of an iterative algorithm for region-by-region image reconstruction. No calibration procedure is needed.

To validate the approach, multi-slice 2D brain imaging data were collected using a T_1-weighted fast gradient echo sequence (FOV 232×232 mm, matrix 232×232, TR/TE 135/4 ms, flip angle 30°) on a 3T clinical MRI scanner. The coil array for data collection had 8 elements uniformly positioned around the anatomy. This gave at most 4 different coil elements in any directions and the maximal parallel imaging acceleration factor was 4. Images were reconstructed from data undersampled manually with a variable density trajectory (Figure 1b). Each 2D dataset had a total of 40 phase encoding lines (including 6 ACS lines), providing a net acceleration factor of 5.8. SAKE+ESPIRiT [1] was used as a reference approach.

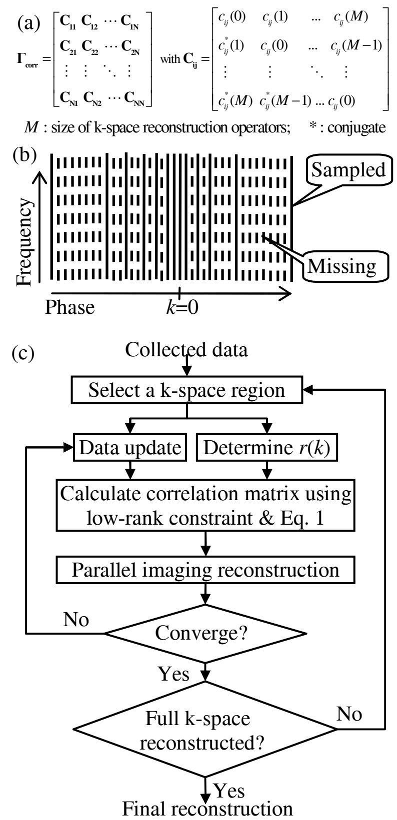

Figure 1. (a)Correlation matrix structure. (b) Variable density undersampling trajectory. (c) Algorithm flow chart.

Results: Figure 2(a) shows the eigenvalues of a few correlation matrices calculated from fully sampled data located in different k-space regions. The correlation matrix from outer k-space data has a considerably lower rank than that from center k-space data. Figures 2(b) and 2(c) show the reconstructed images. The presented approach gives good image quality with considerably lower artifacts than SAKE+ESPIRiT.

Discussion: It should be known that parallel imaging alone is not effective when data are collected with an acceleration factor higher than the coil number in the phase encoding direction. By imposing a low-rank constraint on the correlation matrix in parallel imaging, imaging can be accelerated beyond this limitation. The gain of this approach over SAKE+ESPIRiT arises from the use of a k-space variant constraint. The rank of a correlation matrix is k-space variant because outer k-space data are primarily image details while center k-space data contain most of image contrast information. Image details (e.g. tissue boundary) have higher sparsity in image space than image contrast and thus outer k-space has higher data correlation than center k-space. This data correlation difference generates k-space variation in the rank of correlation matrices, allowing for the use of a higher undersampling factor in outer k-space. It is also found that the presented approach converges fast (<10 iterations).

Conclusion: An additional low-rank constraint on the correlation matrix in parallel imaging reconstruction permits the use of a net acceleration factor higher than the number of coil elements in the phase encoding direction.

References: 1. Shin, P et al., *MRM* 2014, 72: 959-970. 2. Haldar, JP, *IEEE TMI* 2014, 33: 668-681. 3. Li, Y et al., *MRM* 2012, 68:2005-2017.

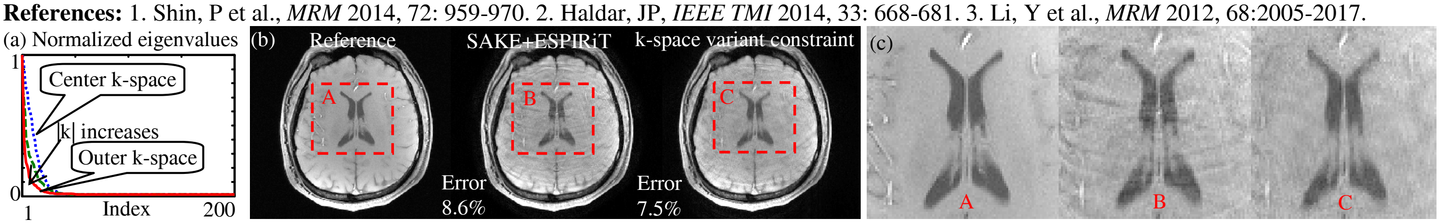

Figure 2.*(a) Plots of eigenvalues (normalized to the first one in each plot) of the correlation matrices calculated from data located in different k-space regions. (b) Reconstructed images using data collected with a net acceleration factor of 5.8. (c) Zoomed images in the red boxes of (b).*