An Image Domain Low Rank Model for Calibrationless Reconstruction of Images with Slowly Varying Phase

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INTRODUCTION: Calibrationless constrained reconstruction methods have attracted recent attention due to their high accuracy and sampling flexibility. Many of these methods are based on low-rank approximations in *k*-space or the image domain. Recently, *k*-space-based methods LORAKS[1-2] and P-LORAKS[3] were proposed for calibrationless reconstruction of images with slowly varying phase from single-channel and multi-channel parallel imaging data. For the same settings, we propose an image domain low-rank model.

THEORY: Let X be a complex image and $\mathrm{R_b}$ be a linear operator that extracts a block (e.g. a small 4×4 patch), indexed by b, from X. For a block with constant phase, the matrix $[\Re(\mathrm{R_b}\{X\})\ \Im(\mathrm{R_b}\{X\})]$ of real and imaginary parts of block b has rank at most half of its column dimension, and inducing low-rankedness in these matrices encourages blocks to have constant phase, thus encouraging slow phase variation over the image. For this purpose, we use a locally low-rank[4] inducing penalty $R(X) = \lambda \sum_b \| [\Re(\mathrm{R_b}\{X\})\ \Im(\mathrm{R_b}\{X\})] \|_*$, where $\| \ \|_*$ is the nuclear norm, λ is a regularization parameter and the sum is taken over disjoint blocks that uniformly tile the image domain. If X is a multi-channel image and channels from block b are represented as columns of $\mathrm{R_b}\{X\}$, the penalty can be viewed as a modification of the penalty $\lambda \sum_b \| \mathrm{R_b}\{X\} \|_*$, previously introduced for image domain calibrationless parallel imaging (CPI) in CLEAR[5] to encourage spatially smooth channel sensitivities.

METHODS: Initial experiments with retrospective under-sampling of a single-channel brain phantom image with a slow phase variation, apart from an artificial phase discontinuity included to evaluate robustness to high frequency phase (Fig. 1a-b), were used for evaluation. Reconstruction with the proposed penalty was implemented by minimizing $\| \mathrm{Y} - \mathrm{FX} \|_F^2 + R(X)$, where F is a Fourier sampling operator and Y is *k*-space data. Contiguous sampling with 9/16 partial k-space was used to compare the method to homodyne.[6] Calibrationless uniform random under-sampling from both halves of k-space was used to compare the method to zero-filling. 8-channel *in vivo* brain data was acquired with a 3D MP-RAGE sequence and under-sampled with 9/16 partial k-space and further with a variable density random (Poisson-disc, R=3) sampling pattern. Image domain CPI was compared to the proposed method, which used the same penalty, only modified to separate real and imaginary parts as described.

RESULTS: Fig. 1b-c shows that reconstruction accuracy of the proposed method is comparable to homodyne for contiguous partial k-space sampling, and both images show artifacts near phase discontinuity. Fig. 1d-e shows accurate reconstruction from calibrationless random sampling with minimal artifacts near the phase discontinuity. Fig. 2 shows that like naïve image domain CPI, the proposed penalty implements parallel imaging, but it recovers both halves of *k*-space, producing sharp root-sum-of-squares images.

DISCUSSION: Images in Fig. 1b-c show that the method can be used with deterministic partial *k*-space sampling. Fig. 1d-e shows that it can also be used with random sampling to exploit slow phase variation where zero-filling and methods using low resolution calibration would fail. Fig. 1d-e also shows robustness to high-frequency phase, which is permitted by the formulation. In compressed sensing applications with random sampling, the penalty term could also be used to complement sparsity constraints. In multi-channel image models, the method simultaneously exploits slow phase variation and performs image domain CPI, but it could also be integrated in other image domain parallel imaging methods. Like image domain CPI, complexity scales with the target image size, and the penalty allows GPU acceleration due to its separability and low memory footprint.

CONCLUSION: We have proposed an image domain locally low-rank model for calibrationless reconstruction of images with slowly varying phase from single and multichannel data. The method can be used to augment other image domain constrained reconstruction methods such as image domain calibrationless parallel imaging.

REFERENCES: [1] Haldar J, TMI, 2014. [2] Haldar J, USC-SIPI Report, 2014. [3] Zhuo J, *et al.* ISMRM 2014 [4] Trzasko J, *et al.* ISMRM 2011. [5] Tzasko J, *et al.* ACSSC, 2011. [6] Noll et al. TMI 1991.

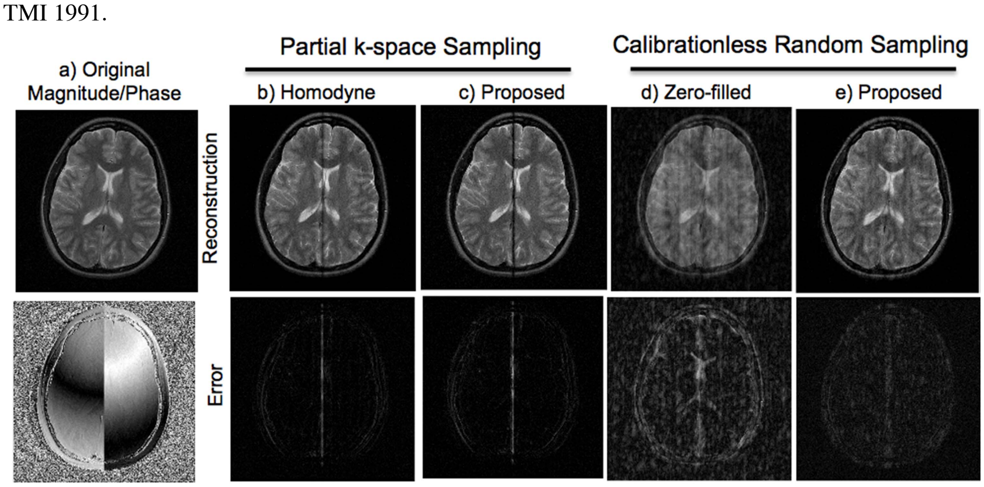

Figure 1: Images reconstructed from conventional partial k-space sampling (b-c) and random symmetric k-space under-sampling (d-e) of single-channel data (a) show high accuracy in both cases and robustness to high-frequency phase.

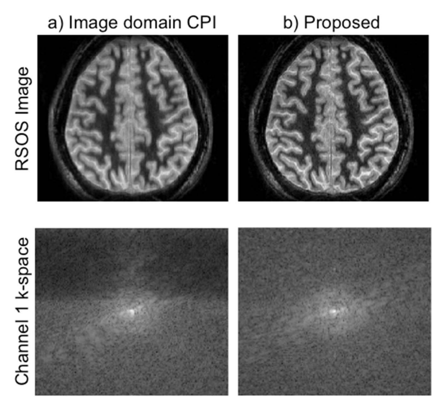

Figure 2: Images reconstructed from calibrationless random partial k-space under-sampling of multi-channel data show that both halves of k-space can be recovered by the proposed method, which separates real and imaginary parts in the locally low-rank formulation.