Pyramidal representation of block Hankel structured low rank matrix (PRESTO) for high perfor mance parallel MRI

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Purpose: In this paper, we propose a novel parallel imaging method called PRESTO (pyramidal representation of block Hankel structure low rank matrix) that do not require any calibration data but still outperform all the existing parallel imaging methods such as GRAPPA [1], SAKE (irregularly sampled k-space without calibration region) [2,3], etc. In multi coil k-space, we reveal that the set of k-space data from several multi coils have novel annihilation properties between different coils as well as within coils. These annihilation properties lead us to a block Hankel structured matrix whose rank should be low dimensional. Accordingly, similar to SAKE, the parallel imaging problem becomes a low rank matrix completion of missing k-space data. However, unlike the SAKE, which exploits the low rankness from all k-space data or needs to combine E-SPIRiT to reduce the complexity, we demonstrate that the low rankness needs to be exploited in a pyramidal representation of block Hankel structured matrix to improve image quality as well as to reduce the complexity. **Theory:** Let $\hat{y}_j$ denote the k-space measurement at the j-th coil $(j = 1,..,C)$ for an underlying unknown image $x(r)$. Now, the annihilation properties we exploit here are different from those of [1-4]. Specifically, it is easy to find the orthogonal image set $q_m(r), m = 1,..,M$ such $q_m(r)x(r) = 0$. For example, if $x(r)$ is a finite supported image, then q_m is an images that is zero on the image support but non-zero at some point at the outside of image support. Moreover, in the parallel imaging, the unknown image $x(r)$ is multiplied with a coil sensitivity map $s_i, i = 1,..,C$. Accordingly, we have the following two types of annihilation properties in k-space:

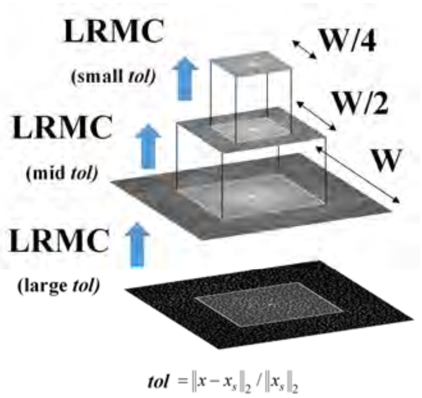


Fig. 1. Overall reconstruction flow of PRESTO. LRMC : Low Rank Matrix Completion. tol : tolerance between estimates and sampled values. x estimated value. x_s : sampled value.

$$\widehat{q_m} * \hat{y}_i = 0, \quad \hat{h}_i * \hat{y}_j - \hat{h}_j * \hat{y}_i = 0, \quad (m = 1,..,M, i, j = 1,..,C) \qquad (1)$$

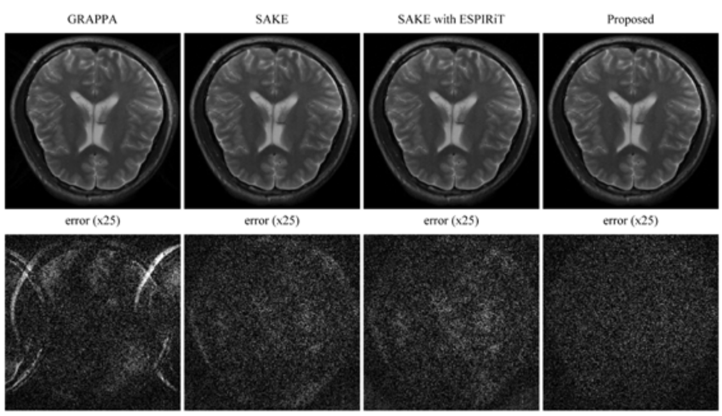
where superscript ^ denotes the corresponding two dimensional Fourier transform. Then, by representing the convolution as multiplication of convolution matrix with a input vector and interchanging the role of the filter and the unknown images, (1) can be represented as

$$[Y_1 \ldots Y_C][Q] = 0, \qquad [Y_1 \ldots Y_C][H] = 0 \qquad (2)$$

where Y_i denotes the block Hankel matrix constructed from i-the k-space measurement, and $[Q]$ denotes the block diagonal matrices with $\hat{q}_i$, and $[H]$ denotes the matrices that combinatorial represents the combination of $\widehat{h_i, h_j, i \neq j}$ at appropriate locations and set zeros at other position. Similar relationship can be obtained for any patch of k-space data by describing the annihilation properties within the patch that are not affected by the boundary of patches. Then, by solving low rank matrix completion problem of missing block Hankel matrix, we can reconstruct original k-space signal without any explicit calculation of filter coefficients from annihilation filters.

To solve the matrix completion problem effectively, we proposed pyramidal representation of the block Hankel matrix. Specifically, at the bottom layer, block Hankel structured matrix from whole k-space is approximated as low rank matrix. After first step, ¼ -size center regions from first step are reconstructed with low rank matrix completion algorithm using the initialization from the previous step. Next iteration proceeds in the same way. One of the main advantages of the proposed pyramidal representation is that we can apply different stopping criterion of matrix completion algorithm at each layer to reflect the signal distribution. Specifically, as high frequency region in k-space domain is highly contaminated with noise compared with low frequency region, we apply early termination at the bottom layer and progressively increase the iteration number for upper layers. This significantly reduces the complexity of the algorithm and further improves the reconstruction quality. As a low rank matrix completion, we use the LMFit[5] combined with ADMM postprocessing step to impose the Hankel structure constraint [6].

Results: The proposed PRESTO algorithm was compared with GRAPPA and SAKE. The k-space data was obtained with Siemens Verio 3T scanner using 2D SE sequence. The acquisition parameters were TR 4000ms, TE 100ms, 256x256 matrix, 6 z-slice with 5mm slice thickness and FOV was 240x240mm. The number of coils was four. All the reconstruction experiments were conducted under retrospective fourth-fold down sampling and use a same uniformly random sampling mask except GRAPPA in which effective down sampling ratio is 3.8 due to existence of the calibration data. NMSE values for GRAPPA, SAKE, SAKE with ESPIRiT and PRESTO are $1.225\text{x}10^{-3}$, $8.921\text{x}10^{-4}$, $9.847\text{x}10^{-4}$ and $7.908\text{x}10^{-4}$, respectively, which showed that proposed algorithm is quantitatively most effective. Moreover, the reconstruction time for PRESTO was 140sec, which is significantly less than SAKE with all k-space data (1213sec). In the difference images, we can see noticeable artifacts in GRAPPA and clustered errors in both type SAKE algorithms, whereas no visible artifact are observed in the result by PRESTO.



Conclusion: PRESTO is a novel parallel imaging algorithm that exploits low rankness in block pyramidal representation of Hankel structured matrix. The new algorithm is based on novel annihilation properties discovered within images and across coils. The pyramidal representation of block Hankel structure matrix fully considered noise distribution in k-space to reduce the reconstruction time as well as to improve the image quality. Our method outperforms all the existing methods even without calibration data.

References: 1. Griswold M., et al. MRM, 2002; 47(6), 1202-1210 2. Shin J., et al. MRM 2014: 72(4). p959-970. 3. Uecker M., et al. MRM, 2014: 71(3) 990-1001. 4. Zhang, et. al. MRM, 2011; 66(5) 1241-1253., 5. Zaiwen W, et al. Math. Prog. Comp., 2012: 333-361. 6. Signoretti M, et al. IEEE conf. on Decision. Control, 2013.