Effective Rank for Automated Parallel Imaging Regularization

Stephen F Cauley[1,2], Kawin Setsompop[1,2], Lawrence Wald[1,2], and Jonathan R Polimeni[1,2]
[1]*Athinoula A. Martinos Center for Biomedical Imaging, MGH/HST, Charlestown, MA, United States,* [2]*Dept. of Radiology, Harvard Medical School, Boston, MA, United States*

TARGET AUDIENCE: Imaging scientists and clinicians interested in stability and accuracy of parallel imaging reconstruction.

PURPOSE: The stable and accurate reconstruction of images from under-sampled data using parallel imaging often requires prior information and/or regularization [1-4]. Regularization of PI based reconstructions can have a significant impact on signal-to-noise and artifact levels, and many attempts have been made to automatically determine the correct balance between image stability and data consistency/low artifact levels [5-7]. We introduce *effective rank* as a proxy for automated PI regularization. Unlike condition number, effective rank correlates with the number of dominate basis vectors that contribute to the image reconstruction. A Line-search can be used to sweep regularization levels to determine the appropriate regularization parameter to meet the desired effective rank. The proposed strategy should be applicable to a wide range of PI reconstruction algorithms and scanner hardware. We demonstrate the benefits of our approach for GRAPPA reconstructions of EPI data with two classes of regularization using 32ch and 52ch head array coils.

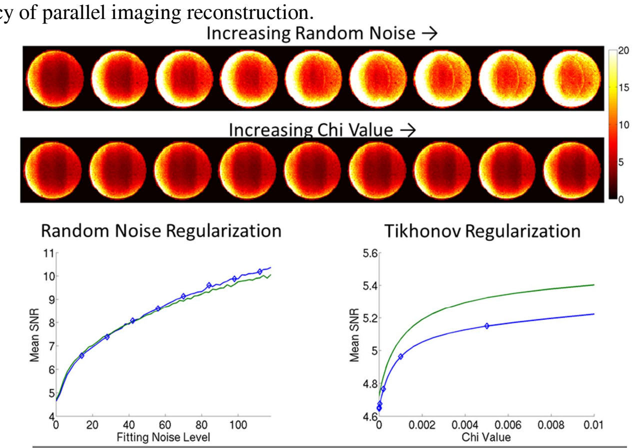

Fig. 1. (top) Image SNR maps for a 52ch GRAPPA reconstruction using random noise or Tikhonov regularization. (bot) Mean SNR increase with increased regularization, plotted for two image slices.

METHOD: GRAPPA reconstruction uses nearest-neighbor k-space weightings across coil channels in order to estimate missing k-space samples from accelerated Cartesian acquisitions. These k-space relationships can be formulated as an over-determined system of linear equations, which can be written compactly as: $AN = B$. During the GRAPPA kernel training phase, A and B are known (supplied by the AutoCalibration Signal or ACS data) and we solve for N through a least-squares formulation. As new accelerated data arrives, it is assembled into new undersampled matrix $\tilde{A}$ and the kernels N are applied to calculate the missing samples contained within the resulting matrix $\tilde{B}$. The stability of the training matrix A, and the related stability of the kernels N, are both critically important to the performance of GRAPPA reconstruction. A simple metric for stability would be the condition number of the kernel matrix. Here, the singular value decomposition could be used to factor $N = USV^H$, where the singular values are in descending order $S = diag\{\sigma_{max}, \dots, \sigma_{min}\}$. Regularization is added until a desired condition number $\tilde{\kappa} = \tilde{\sigma}_{max}/\tilde{\sigma}_{min}$ is reached. However, condition number only describes the worst-case error propagation and this metric does not correlate well with image SNR and artifact levels; see Figs. 1 and 2. Alternative approaches simulate data to test image reconstruction accuracy/stability in order to arrive at a maximum likelihood regularization estimate. In this work, we avoid costly simulations and define a rank proxy to determine appropriate levels of regularization. First, a relative threshold cut-off on the singular value decay is defined from the non-regularized kernels: $e_{trunk} = (\sigma_{median} - \sigma_{min})/(\sigma_{max} - \sigma_{min})$. Given a regularization level, the number singular values that satisfy $\tilde{\sigma}_i > e_{trunk}\tilde{\sigma}_{max}$ determines the effective rank. If only small changes in the effective rank are observed as the regularization level is increased, there will be diminishing benefits to further regularization and this can lead to increased image artifact levels.

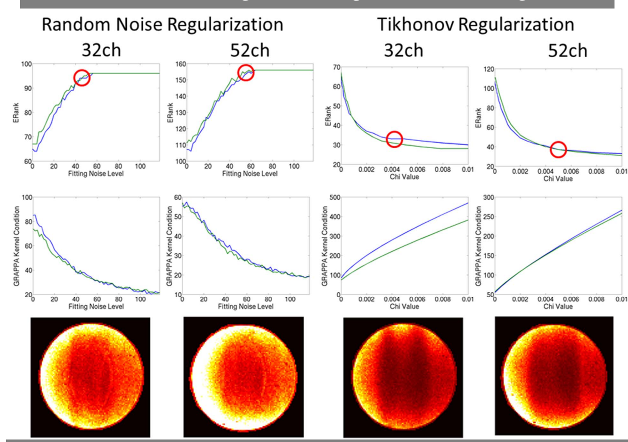

Fig. 2. (top) Effective rank for 32ch and 52ch GRAPPA reconstructions using random noise Tikhonov regularization for two slices. (mid) Kernel condition numbers are shown across regularization levels. (bot) SNR maps corresponding to regularization picked from effective rank trends.

RESULTS: The application of the effective rank proxy was demonstrated with a spherical phantom on a 3 T Siemens Skyra scanner using both Siemens 32ch and 52ch head array coils. There were 32 single-shot gradient-echo EPI slices acquired at 2 mm isotropic resolution with matrix size of 96×96, TE=30 ms, TR=2 s, flip angle 90°, and in-plane acceleration factor of 4 with 88 autocalibration lines. We consider both the standard Tikhonov regularization with new normal equation matrix $\left(A^H A + \chi * mean\left(tr(A^H A)\right) * I\right)$ and Gaussian noise regularization which instead adds a random matrix. Fig. 1 shows SNR maps of the 52ch reconstructions across 7 different regularization settings; the corresponding values are indicated on the Mean SNR plots (Fig. 1 bot). Fig. 2. shows the effective rank trends for both regularization approaches and coils. In addition, the condition number trend for the associated GRAPPA kernels is shown below. Finally, the clear plateau observed in each effective rank trend can be used to choose a regularization level that may be used to balance the SNR and artifact level (circled locations); the corresponding SNR maps are provided.

DISCUSSION: Effective rank can be used as a proxy to successfully balance SNR and artifact levels for automated regularization of PI reconstruction. As can be seen from Fig. 2 effective rank accurately predicts the transition point from improved SNR to over-regularization/increased artifacts (no such transition is observable with the condition number). This distinct transition point in the erank trend can be used as a reference point to allow for image reconstructions favoring either increased SNR or decreased artifact level. The method avoids costly simulations and only requires evaluation of the singular value decay rates at progressively higher regularization rates. For standard kernel sizes and array coils, direct SVD methods will allow for efficient regularization parameter estimation. In the case of higher acceleration factors and large array coils, matrix compression techniques [8] can allow for efficient approximation of the effective rank through divide-and-conquer based eigenvalue techniques [9].

REFERENCES: [1] Pruessmann et al. MRM 1999; [2] Griswold, et al. MRM 2002; [3] Lustig, et al. MRM 2010; [4] Lustig, et al. MRM 2007; [5] Ying et al. IEEE EMBS 2004; [6] Lin et al. MRM 2004; [7] Weller et al. MRM 2014; [8] Xia, et al. Numer. Linear Algebra Appl., 2010; [9] Nakatsukasa et al. SIAM J. Sci. Comput. 2013; **SUPPORT:** R01EB006847, R00EB012107, R01EB017219, P41RR014075, K01-EB011498, and U01MH093765.