Balanced sparse MRI model: Bridge the analysis and synthesis sparse models in compressed sensing MRI

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Introduction: Compressed sensing (CS) has shown to be promising to accelerate magnetic resonance imaging (MRI) [1]. In CS, MR images are usually reconstructed by synthesis or analysis reconstruction models based on different assumptions [2-4]. The synthesis model assumes that an image is a sparse combination of atom signals, while the analysis model assumes that an image is sparse after the application of an analysis operator [4]. These two sparse models are obviously different under a redundant tight frame system. Balanced model is a new sparse model that bridges analysis and synthesis models [5]. In this work, we study the performance of the balanced model in CS-MRI for different tight frames. It is found that, adjusting a balance parameter will approach to both typical sparse models as well as the transition performance between them.

Method: The analysis, balance and synthesis sparse models in CS-MRI are as follows

Balance: $\hat{\mathbf{x}} := \mathbf{\Psi}^{*}\hat{\boldsymbol{\alpha}};\ \ \hat{\boldsymbol{\alpha}} := \arg\min_{\boldsymbol{\alpha}} \left\{ \frac{1}{2}\left\|\mathbf{y} - \mathbf{F}_{\mathrm{U}}\mathbf{\Psi}^{*}\boldsymbol{\alpha}\right\|_2^2 + \lambda\left\|\boldsymbol{\alpha}\right\|_1 + \frac{\beta}{2}\left\|(\mathbf{I} - \mathbf{\Psi}\mathbf{\Psi}^{*})\boldsymbol{\alpha}\right\|_2^2 \right\}$

$\beta \to +\infty$ → Analysis: $\hat{\mathbf{x}} := \arg\min_{\mathbf{x}} \frac{1}{2}\left\|\mathbf{y} - \mathbf{F}_{\mathrm{U}}\mathbf{x}\right\|_2^2 + \lambda\left\|\mathbf{\Psi}\mathbf{x}\right\|_1$

$\beta = 0$ → Synthesis: $\hat{\mathbf{x}} := \mathbf{\Psi}^{*}\hat{\boldsymbol{\alpha}};\ \ \hat{\boldsymbol{\alpha}} := \arg\min_{\boldsymbol{\alpha}} \frac{1}{2}\left\|\mathbf{y} - \mathbf{F}_{\mathrm{U}}\mathbf{\Psi}^{*}\boldsymbol{\alpha}\right\|_2^2 + \lambda\left\|\boldsymbol{\alpha}\right\|_1$

where $\mathbf{x} \in \mathbb{C}^{N\times 1}$ denotes the MR image concatenated to a vector. $\boldsymbol{\alpha} \in \mathbb{C}^{Q\times 1}$ denotes the coefficients. $\mathbf{\Psi} \in \mathbb{C}^{Q\times N}\,(Q > N)$ is the analysis operator of a tight frame. $\mathbf{F}_{\mathrm{U}} \in \mathbb{C}^{M\times N}\,(M < N)$ is the Fourier undersampling operator and $\mathbf{y} \in \mathbb{C}^{M\times 1}$ is the undersampled k-space data. λ is a trade-off parameter between sparsity and data fidelity and $\gamma = \frac{1}{1+\beta}$ is the balancing parameter controlling the balance sparse model to achieve the solutions of the analysis model, synthesis model and some in between called balance model. $\gamma = 0.5$ is set as called as a typical balance model.

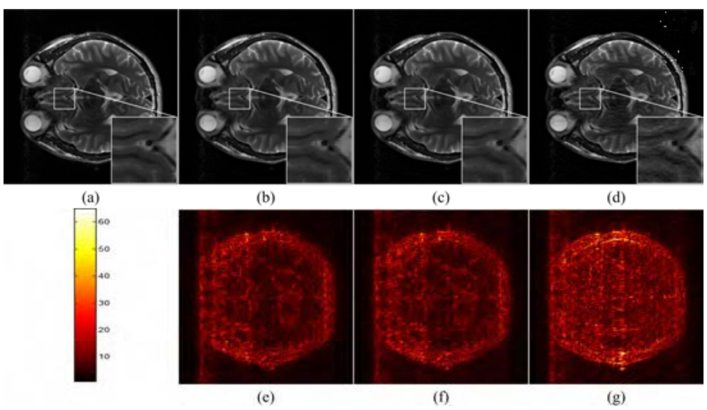

Fig.1 Reconstructed images of analysis, balance and synthesis models using the tight frame PBDW. (a) the fully sampled image; (b)-(d) are reconstructed images using analysis, balanced and synthesis models, respectively; (e)-(g) are 6 times scaled reconstruction errors for images in (b)-(d), respectively. The RLNEs for (b)-(d) are 0.085, 0.086 and 0.114.

Results: The analysis operator of the tight frames used in the experiments are patch-based directional wavelet (PBDW) [2], contourlet [6], and a translation invariant discrete cosine transform (TIDCT) [1]. The brain image of size 256 _ 256is acquired from a healthy volunteer at a 3T Siemens Trio Tim MRI scanner using the T2-weighted turbo spin echo sequence (TR/TE = 6100=99 ms, FOV = 220_220 mm2, slice thickness = 3 mm). And the ratio of sampled k-space data is 40% with down sampling only along the phase encoding direction. The relative L2 norm error , $\mathrm{RLNE} := \left\|\hat{\mathbf{x}} - \mathbf{x}\right\|_2 / \left\|\mathbf{x}\right\|_2$, is used to assess the reconstruction numerically where $\mathbf{x}$ denotes the ground truth image and $\hat{\mathbf{x}}$ denotes the reconstructed image.

Conclusions: By tuning the balancing parameter, the balance model achieves the solutions of three models. It is found that the typical balance model ($\beta = 1$ or $\gamma = 1/2$) has a comparable performance with the analysis model. Besides, both of them achieve lower reconstructed errors than the synthesis model no matter what value the balancing parameter is. This observation is consistent when experiments are done with different amount of undersampled k-space data. Results show that the proposed algorithm can provide a unified framework to explore the performance of the three sparse models.

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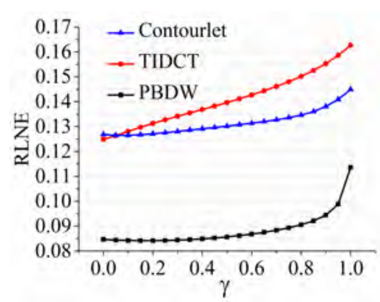

Fig.2 Reconstructed errors versus balancing parameter for different tight frames: Contourlet, translation invariant discrete cosine transform (TIDCT) and patch-based directional wavelets (PBDW).

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