Double Smoothing Method-based Algorithm for MR Image Reconstruction with Partial Fourier Data

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Target Audience: Engineers who are interested in MR image reconstruction

Purpose: Sparse MRI techniques, which employ the Compressed Sensing (CS) theory to exploit sparsity implicit in MR images are promising to improve the speed of MRI. It is still challenging to obtain an accurate reconstruction from highly undersampled MRI data. In this work, we developed a novel sparse MRI algorithm based on Nesterov's smoothing scheme[1], which.utilizes a proximal function to smooth the non-smooth *l*1 regularization terms.

Theory: According to the mathematical theory of CS, an MR image, which is generally sparse in certain transform domain, can be recovered from a reduced amount of randomly sampled k-space data, provided an appropriate nonlinear recovery scheme is used. The reconstruction is obtained by solving the following canonical constrained optimization problem: $\min_m \|Wm\|_1 + \rho\|Dm\|_1$, s.t. $\|F_u m - y\|_2 \le \varepsilon$, where vector m denotes the image to be reconstructed, y denotes the measured k-space data, W represents the linear sparsifying transform matrix, D represents the finite difference operator, F_u denotes the undersampled Fourier transform, ρ is a positive parameter trading-off between the two sparsity constraints, and ε controls the fidelity of the reconstruction to the measured data.

To solve the above minimization, the proposed algorithm mainly consists of four procedures: 1), determine the equivalent conic formulation; 2), determine its dual; 3), apply smoothing; and 4), solve using an optimal first-order method. Each procedure of the proposed algorithm is described as follows[3].

First, according to [3], the conic formulation of the above minimization problem is: $\min_m \; w + \rho d$ s.t. $\|Wm\|_1 \le w$, $\|Dm\|_1 \le d$, $\|F_u m - y\|_2 \le \varepsilon$, where w and d are new scalars.

Second, inspired by Nesterov[2], the dual variable λ can be partitioned as $\lambda = (x^{(1)}, \gamma^{(1)}, x^{(2)}, \gamma^{(2)}, x^{(3)}, \gamma^{(3)})$, where $\|x^{(1)}\|_\infty \le \gamma^{(1)}, \|x^{(2)}\|_\infty \le \gamma^{(2)}, \|x^{(3)}\|_2 \le \gamma^{(3)}$. The Lagrangian is given by: $\mathrm{L}(m, w, d; x^{(1)}, \gamma^{(1)}, x^{(2)}, \gamma^{(2)}, x^{(3)}, \gamma^{(3)}) = w + \rho d - < x^{(1)}, Wm > - \gamma^{(1)} w - \rho < x^{(2)}, Dm > - \gamma^{(2)} \rho d - < x^{(3)}, y - F_u m > - \varepsilon \gamma^{(3)}$, setting $\gamma^{(1)} = \gamma^{(2)} = 1$ to ensure the Lagrangian is bounded, therefore the simplified dual problem is: $\max_m \; < y, x^{(3)} > - \varepsilon \|x^{(3)}\|_2$ s. t. $\mathrm{F}_u^* x^{(3)} - W^* x^{(1)} - \rho D^* x^{(2)} = 0, \|x^{(1)}\|_\infty \le 1, \|x^{(2)}\|_\infty \le 1.$

Third, we use a standard proximity function $p(m) = \frac{1}{2}\|m - m_0\|^2$ to apply smoothing to the above dual problem. The smoothed dual function becomes:

$$g_\mu(x^{(1)}, x^{(2)}, x^{(3)}) = \inf_m \frac{1}{2}\mu\|m - m_0\|_2^2 - < x^{(1)}, Wm > - \rho < x^{(2)}, Dm > - < x^{(3)}, y - F_u m > - \varepsilon \|x^{(3)}\|.$$

Fourth, the solution to the smoothed dual problem is given by:

$$m(x) = m_0 + \mu^{-1}(\mathrm{W}^* x^{(1)} + \rho \mathrm{D}^* x^{(2)} - F_u^* x^{(3)}),\; x_k^{(1)} = Trunc(y_k^{(1)} - \theta_k^{-1} t_k^{(1)} m_*^{(1)}, \theta_k^{-1} t_k^{(1)}),\; x_k^{(2)} = CTrunc(y_k^{(2)} - \theta_k^{-1} t_k^{(2)} \rho m_*^{(2)}, \theta_k^{-1} t_k^{(2)}),\; x_k^{(3)} = Shrink(y_k^{(3)} - \theta_k^{-1} t_k^{(3)} m_*^{(3)}, \theta_k^{-1} t_k^{(3)} \varepsilon),$$

Where $m_* = (\mathrm{W}\, m(x), Dm(x), y - F_u m(x))$, $Trunc(a,b) = \mathrm{sgn}(a) \cdot \min\{|a|, b\}$, $[CTrunc(a,b)]_k = \min\{1, b/|a_k|\} \cdot a_k$ is a complex-valued version of $Trunc$, $Shrink(a,b) = \max\{1 - b/\|a\|_2, 0\} \cdot a$.

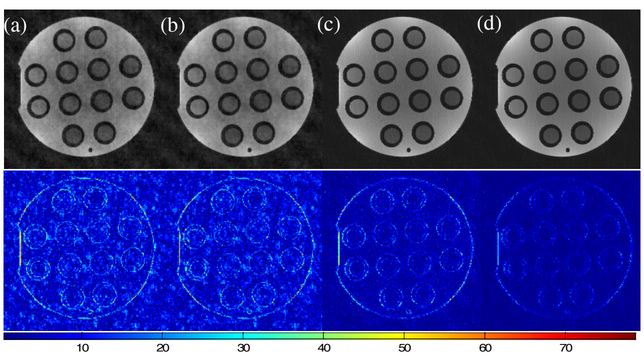

Fig. 1. Phantom images reconstructed from 20% samples. Top row: reconstruction by (a) TVCMRI, (b) RecPF, (c) FCSA and (d) our method. Bottom row: error maps.

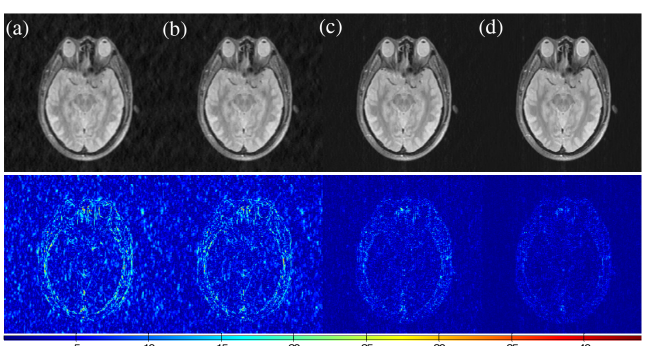

Fig. 2. In vivo brain MR images reconstructed from 20% sampling. Top row: reconstruction by (a) TVCMRI, (b) RecPF, (c) FCSA and (d) our method. Bottom row: error maps.

Methods: We compare the performance of TVCMRI[4], RecPF[5] and FCSA[6] with our proposed algorithm on phantom and in vivo brain MRI data with resolution 256 x 256. In the experiments, the k-space data is retrospectively undersampled according to the following manner. We randomly obtain more samples in lower frequencies and less samples in higher frequencies[6]. All experiments are conducted on a 2.33GHz PCin Matlab v7.13 (R2011b) environment.In [4~6], the regularization parameters are set as 0.001 and 0.035, for fair comparisons, the regularization parameter ρ is set as 0.029.

Results: Figure 1-2 show the visual comparisons of the reconstruction and the corresponding error maps by different algorithms. Our method outperform the othres for phantom and in vivo data. Figure 3 gives the qualitative comparisons between different methods in terms of PSNR over different sampling ratios.

Conclusion: Experimental results demonstrate that the reconstruction algorithm we have presented tailored for MRI, which demonstrated in the phantom and in-vivo dataset an improvement in image quality and reconstruction accurate with respect of some state-of-the-art reconstruction algorithms.

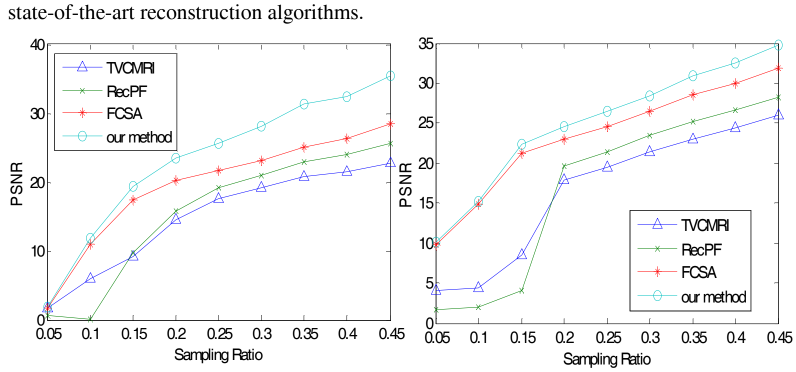

Fig. 3. PSNR for several algorithms for (left) phantom data and (right) fat brain in vivo data with different sampling ratios.

References:[1] Y. Nesterov, Math. Prog., Ser. A, 103:127-152,2005. [2] Y. Nesterov, Math. Prog., Ser. B, 140:125-161,2013. [3] Stephen Becker et al., Math. Prog. Comp., 3:165-218, 2011.[4] Ma et al., IEEE CVPR, pp. 1-8, 2008. [5] Yang et al., IEEE J-STSP, 4, 288-297, 2010. [6] Huang et al., MIA,15(5), 670-679, 2011.