Graph-based compressed sensing MRI image reconstruction: View image patch as a vertex on graph

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INTRODUCTION: Compressed sensing (CS) accelerates magnetic resonance (MR) imaging [1] by under-sampling the k-space data. Although undersampling will introduce aliasing artifacts, high quality images can be reconstructed by enforcing a sparse regularity on them.The sparse representation plays a key role in CS. Previous methods are often based on pre-constructed bases [2], trained dictionaries [3] or optimized sparsifying transforms [4,5]. In this work, an emerging graphical model is constructed to capture geometry and structure of a MR image which transform it more regular. Then, wavelet transform is applied on the re-ordered pixels to sparsify the image.

METHOD: By breaking an image into overlapped patches and denoting the patches as vertexes and Euclidean distances between patches as edges, a patch-based graph can be constructed. Fig. 1 shows the graph of several patches. Smaller Euclidean distance implies more similarity between the centered pixels. To rearrange pixels to be a smooth signal, a traveling sales man problem is to be solved to find the optimal rearrangement [6]. The corresponding permutation matrix $\mathbf{P}^{\mathbf{T}}$ is applied to pixels before wavelet filtering, which can be expressed as $\mathbf{\Psi}^{\mathrm{T}}\mathbf{P}^{\mathrm{T}}$.The same reordering and filtering procedures can be applied to pixels of other pixels in a patch without increasing difficulty. When the size of an column-wise patch is J, we use $\mathbf{R}_j, j=1,\cdots,J$ to denote the reordering operator. The ℓ_1 norm is adopted to enforce the sparsity of the coefficients and the image reconstruction problem is:

$$\min_{\mathbf{x}} \left\{ \sum_{j=1}^{J} \left\| (\mathbf{P\Psi})^T \mathbf{R}_j \mathbf{x} \right\|_1 + \frac{\lambda}{2} \left\| \mathbf{y} - \mathbf{F}_U \mathbf{x} \right\|_2^2 \right\} \quad (1)$$

where λ is the regularization parameter that trades data fidelity off the sparsity. In this paper, the initial guide image is obtained by using shift-invariant discrete wavelet (SIDWT) which can mitigate blocky artifacts introduced by conventional orthogonal discrete wavelets [4,5].

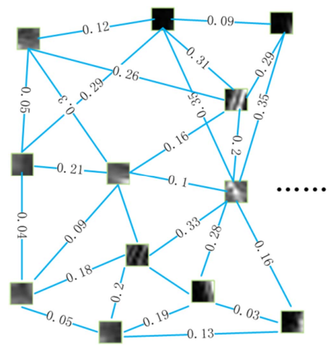
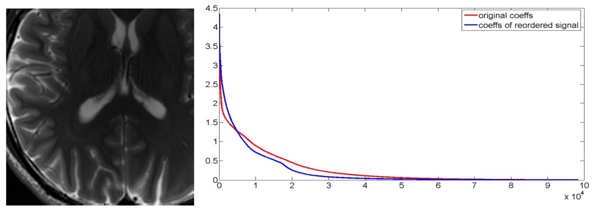


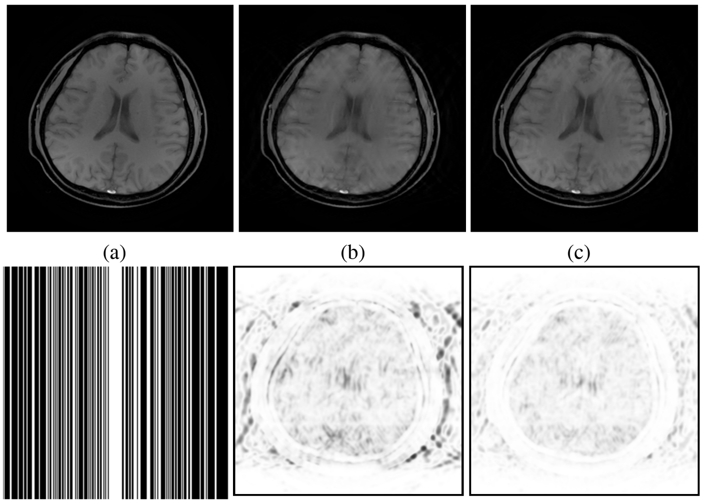
Fig. 1 A patch-based graph of a MR image.



(a) (b)

Fig. 2 Sparsity analysis of the proposed reordering and filter scheme.

RESULTS: By incorporating the graph structure in image, a reordering procedure can make a signal regular as shown in Fig. 2. Coefficients after reordering drop faster than that of original signal, which makes the sparse representation more efficiency. In the reconstruction, the relative ℓ_2 norm error (RLNE) and structure similarity index (SSIM) [7] are employed to evaluate the reconstruction error in this work. RLNE is defined as $e(\hat{\mathbf{x}}) = \|\hat{\mathbf{x}} - \tilde{\mathbf{x}}\| / \|\tilde{\mathbf{x}}\|$, where $\hat{\mathbf{x}}$ is the reconstructed image and $\tilde{\mathbf{x}}$ is the ground truth. SSIM measures structure similarity of two corresponding local windows. A smaller RLNE implies more consistent between the reconstructed image and the full sampled image, while Higher MSSIM values indicate stronger detail preservation in reconstruction. Fig. 2 shows the reconstructed images. The reconstruction error, RLNE, image structure quality, mean SSIM, are in its captions. The graph-based sparse representation leads to lower reconstruction error and better image structure quality.



(a) (b) (c)

(d) (e) (f)

Fig. 3 Reconstructed images. (a) full sampled image. (b) and (c) are reconstructed images using SIDWT and the proposed method, (d) the undersampling pattern with 35% data, (e) and (f) are the SSIMs corresponding to (b) and (c). The RLNE of (b) and (c) are 0.103 and 0.065, the mean SSIM of (b) and (c) are 0.9319 and 0.9629.

CONCLUSIONS: A graph-based compressed sensing MRI image reconstruction is proposed. This method views an image patch as a vertex on graph and reorders the pixel to be smooth by traveling this graph with shortest path. Image reconstruciong from compressively sampled data shows that the proposed reconstruction method outperforms conventional wavelets in terms of visual quality and evaluation criteria.

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