A fast algorithm for tight frame-based nonlocal transform in compressed sensing MRI

Xiaobo Qu[1], Yunsong Liu[1], Jing Ye[1], Di Guo[2], Zhifang Zhan[1], and Zhong Chen[1]

[1]*Department of Electronic Science, Xiamen University, Xiamen, Fujian, China,* [2]*School of Computer and Information Engineering, Xiamen University of Technology, Xiamen, Fujian, China*

Introduction: Compressed sensing magnetic resonance imaging (CS-MRI) is to reconstruct magnetic resonance (MR) images from undersampled k-space data [1]. The rationale behind CS-MRI is that MR images are sparse in some transform domain. The quality of the reconstructed images highly depend on the sparsify capability of that transform [2]. Previously, the self-similarity of images has been proven to be very effective in image denoising [3] and image reconstruction [4]. Patch-based nonlocal operator (PANO) [3] was proposed as a linear operator to exploit the nonlocal self-similarity of MR images to further sparsify images and has shown advantages in edge preserving. However, the original PANO is a frame and its numerical algorithm for CS-MRI problem is solved by the alternating direction minimization with continuation (ADMC) [4]. These two aspects lead the reconstruction to be time consuming. In this work, we first convert the PANO into a tight frame, and then applied the alternating direction method of multipliers (ADMM) [5] algorithm to accelerate the image reconstruction.

Method: Patch-based nonlocal operator for the j^{th} group of patches is mathematically represented as $\mathbf{\Psi}_j = \mathbf{\Psi}_{3D}\mathbf{R}_{v_j}\mathbf{P}_j$, where $\mathbf{P}_j$ extracts patches for the j^{th} group, $\mathbf{R}_{v_j}$ reorders the patches in this group according to the index contained in v_j, $\mathbf{\Psi}_{3D}$ denotes the 3D Haar wavelet transform to the j^{th} group of patches. Due to the overlap during the extraction of patches, we have the following relation of PANO $\sum_j \mathbf{\Psi}_j^*\mathbf{\Psi}_j = \mathbf{\Lambda}$, where $\mathbf{\Lambda}$ is diagonal with diagonal elements are overlap factor of each pixel and the different overlap factor of pixels makes PANO a frame. However, after a simple operation as $\mathbf{\Phi}_j = \mathbf{\Psi}_j\left(\sqrt{\mathbf{\Lambda}}\right)^{-1}$ we can get the relation that $\sum_j \mathbf{\Phi}_j^*\mathbf{\Phi}_j = \mathbf{I}$. In other words, this operation converts PANO into a tight frame.

PANO based CS-MRI is to solve

$$\min_{\mathbf{x}} \sum_{j=1}^{J} \left\|\mathbf{\Psi}_j\mathbf{x}\right\|_1 + \frac{\lambda}{2}\left\|\mathbf{y} - \mathbf{F}_U\mathbf{x}\right\|_2^2 ,$$

where $\mathbf{x} \in \mathbb{C}^{N\times 1}$ denotes the MR image, $\mathbf{F}_U \in \mathbb{C}^{M\times N}\left(M < N\right)$ is the Fourier undersampling operator and $\mathbf{y} \in \mathbb{C}^{M\times 1}$ is the undersampled k-space data. λ is a trade-off parameter between sparsity and data fidelity. The ADMM algorithm is summarized in Table 1.

Table 1. ADMM algorithm for PANO based CS-MRI

Input $\mathbf{x}_0, \lambda, \rho, \mu$ and initialize $\mathbf{d}_0 = \mathbf{0}, \mathbf{h}_0 = \mathbf{0}$
Repeat until converge $\boldsymbol{\alpha}_{n+1}^j = T_{\lambda/\rho}\left(\mathbf{\Psi}_j\mathbf{x}_n - \mathbf{d}_n^j\right)$ $(\mu\mathbf{F}_U^*\mathbf{F}_U + \rho\sum_j \mathbf{\Psi}_j^*\mathbf{\Psi}_j)\mathbf{x}_{n+1} = \left(\mu\mathbf{F}_U^*\left(\mathbf{y} + \mathbf{h}_n\right) + \rho\sum_j \mathbf{\Psi}_j^*\left(\boldsymbol{\alpha}_n + \mathbf{d}_n\right)\right)$ $\mathbf{h}_{n+1} = \mathbf{h}_n - \left(\mathbf{F}_U\mathbf{x}_{n+1} - \mathbf{y}\right)$ $\mathbf{d}_{n+1}^j = \mathbf{d}_n^j - \left(\mathbf{\Psi}_j\mathbf{x}_{n+1} - \boldsymbol{\alpha}_{n+1}^j\right)$
Output $\mathbf{x}$

Results: 40% of fully sampled k-space data(with size 256×256) along the phase encoding direction is used in reconstruction. The relative l_2 norm error (RLNE) is used to assess the reconstruction performance numerically and is defined as $\text{RLNE} := \left\|\hat{\mathbf{x}} - \mathbf{x}\right\|_2 / \left\|\mathbf{x}\right\|_2$, where $\mathbf{x}$ denotes the ground truth image and $\hat{\mathbf{x}}$ denotes the reconstructed image. The compuation time versus the reconstruction is reported in Fig. 1. Compared to the original frame version of PANO, its tight frame can significanlty speed up both the ADMC and ADMM algorithms in CS-MRI without increasing the reconstruction errors. Besides, the ADMM algorithm converges much faster than the previously used ADMC algorithm.

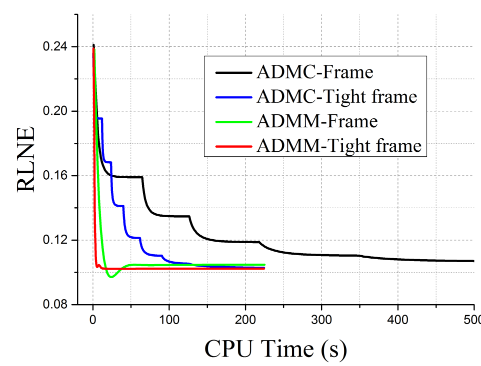

Fig. 1. Empirical convergence of different approaches.

Conclusions: Patch-based nonlocal operator (PANO) can exploit the nonlocal self-similarity of MR images and have shown advantage in edge preserving. A tight frame of PANO is proposed and incorporated into the alternating direction method of multipliers algorithm to significantly accelerate the image reconstruction in compressed sensing MRI.

Acknowledgement: This work was partially supported by the NNSF of China (61201045, 11375147 and 61302174) and Fundamental Research Funds for the Central Universities (2013SH002).

References:

1. Lustig, M., *et al.*, Sparse MRI: The application of compressed sensing for rapid MR imaging. ***Magnetic Resonance in Medicine***. 2007; 58(6): 1182-1195.
2. Qu, X., *et al.*, Undersampled MRI reconstruction with patch-based directional wavelets. ***Magnetic Resonance Imaging***. 2012; 30(7): 964-977.
3. Dabov, K., *et al.* , Image denoising by sparse 3-D transform-domain collaborative filtering. ***IEEE Transactions on Image Processing***, 2007, 16(8): 2080-2095.
4. Akcakaya, M., *et al.*, Low-dimensional-structure self-learning and thresholding: Regularization beyond compressed sensing for MRI reconstruction. ***Magnetic Resonance in Medicine***, 2011, 66(3): 756-767.
5. Qu, X., *et al.*, Magnetic resonance image reconstruction from undersampled measurements using a patch-based nonlocal operator. ***Medical Image Analysis***. 2014; 18(6): 843-856.
6. Yang, J.-F., *et al.*, A fast TVL1-L2 minimization algorithm for signal reconstruction from partial Fourier data. Report. 2008; CAAM, Rice Univ.
7. Afonso, M. V., *et al.*, An augmented lagrangian approach to the constrained optimization formulation of imaging inverse problems. ***IEEE Transactions on Imaging Processing.*** 2011; 20(3):681-695