Extended Phase Graphs: Understanding a Common Misconception of the Framework which Leads to the Failure of Programming It Correctly

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Purpose

The extended phase graph (EPG) concept represents a neat and favorite approach for depicting and understanding the magnetization response of a broad variety of MR sequences. However, users often fail at actually coding the framework into properly working software. From the experience of the author, one major reason is a common misconception that is linked to the complex spatial Fourier transforms of magnetization, on which the concept of EPGs is based. Understanding this intricacy immediately leads to the solution how EPGs have to be coded - and how possibly existing software of a user has to be adapted.

Outline of Content

The foundation of the EPG concept is correctly represented by a set of three base functions as shown in the box Eq. 1 (1D formulation). The longitudinal part $\tilde{Z}(k)$ will be omitted in the following, because it is not important for the present discussion. Based on the original notion by Woessner that a 180° refocusing pulse converts dephasing transverse M_x+iM_y magnetization into rephasing M_x+iM_y magnetization, $\tilde{F}_+(k)$ is usually understood and labelled as the dephasing and $\tilde{F}_-(k)$ as the rephasing transverse component within the EPG concept. Since an RF pulse inverts the sign of the dephasing coordinate k of a given EPG population $\tilde{F}(k)$, users frequently, therefore, tend to take an $\tilde{F}(+k)$ as the dephasing component and an $\tilde{F}(-k)$ as the rephasing component. Admittedly, using the sign of k as a distinction is of high practical value and it also seems in accordance with the EPG diagrams as shown in Fig. 1. Then, in a straight forward way, the EPG concept is often coded using the RF pulse transfer matrix from Eq. 2, implying that $\tilde{F}_+ = \tilde{F}(+k)$ and $\tilde{F}_- = \tilde{F}(-k)$. Yet, this procedure will cause the software to give wrong results. *Why*?

$$\tilde{F}_+(k) = \int_V \left(M_x(r) + iM_y(r)\right) e^{-ikr}\, d^3r$$
$$\tilde{F}_-(k) = \int_V \left(M_x(r) - iM_y(r)\right) e^{-ikr}\, d^3r$$
$$\tilde{Z}(k) = \int_V M_z(r)\, e^{-ikr}\, d^3r$$

Equation 1: Full base functions of the EPG.

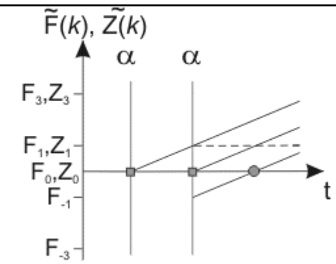

Figure 1: Example for a simple EPG diagram, where negative k denote rephasing transverse pathways.

First, as can be seen from the base functions (Eq. 1), there is not a single $\tilde{F}(\pm k)$ but there are two transverse base functions that actually span over the full range of k: $\tilde{F}_+(\pm k)$ and $\tilde{F}_-(\pm k)$! However, both $\tilde{F}$ "domains" are not independent; they are linked by the symmetry relation $\left(\tilde{F}_+(k)\right)^* = \tilde{F}_-(-k)$, because the complex Fourier transform in Eq. 1 converts two former real magnetization components M_x and M_y into two complex magnetization states (four values). Thus, each of the domains $\tilde{F}_+$ and $\tilde{F}_-$ already contains the full information of, basically, the "EPG idea" - they are merely anti-symmetrically complex conjugated to each other (see above).

$$\begin{bmatrix}\tilde{F}_+\\ \tilde{F}_-\\ \tilde{Z}\end{bmatrix}^+ = \begin{bmatrix}\cos^2\frac{\alpha}{2} & e^{2i\Phi}\sin^2\frac{\alpha}{2} & -ie^{i\Phi}\sin\alpha\\ e^{-2i\Phi}\sin^2\frac{\alpha}{2} & \cos^2\frac{\alpha}{2} & ie^{-i\Phi}\sin\alpha\\ -\frac{i}{2}e^{-i\Phi}\sin\alpha & \frac{i}{2}e^{i\Phi}\sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\tilde{F}_+\\ \tilde{F}_-\\ \tilde{Z}\end{bmatrix}^-$$

Equation 2: EPG transfer matrix representing an RF pulse.

Second, the RF pulse transfer matrix (Eq. 2) includes the intrinsic facts that an RF pulse not only inverts the sign of k of a given $\tilde{F}_\pm(k)$, but also complex conjugates the population, i.e. it exchanges populations between the two domains $\tilde{F}_+$ and $\tilde{F}_-$. On the contrary, gradients merely introduce dephasing and rephasing, represented by a change of k by Δk of a given state $\tilde{F}_+(k)$ or $\tilde{F}_-(k)$. Therefore, a gradient does not change the $\tilde{F}$ domain of a given state, it preserves it.

As a result, directly using Eq. 2 implies that all $\tilde{F}(+k)$ are $\tilde{F}_+(+k)$ and all $\tilde{F}(-k)$ belong to the complex conjugated $\tilde{F}_-$ domain in one's EPG coding. This presumption does not directly represent an error or necessarily a misconception of the EPG framework; however, it causes both complex conjugated domains to be so to say "stitched together" at k=0 in the code. Thus, the crux is that the programmer has to define whether the important $\tilde{F}(k=0)$ state – representing echoes and FIDs – is an $\tilde{F}_+(0)$ or an $\tilde{F}_-(0) = \left(\tilde{F}_+(0)\right)^*$ state. Furthermore, if a gradient causes a transition from an $\tilde{F}(+k)$ to an $\tilde{F}(-k)$ state or vice versa, it will be in the "wrong domain" regarding the EPG coding, because of the lacking domain change by the gradient. Not taking care of both intricacies will lead to the failure of the coded EPG.

What can be done? Two solutions.

(1) Defining $\tilde{F}(0)$ as an $\tilde{F}_+(0)$ component, for instance, and complex conjugate all populations of the states that transition from $\tilde{F}_-(k<0)$ to $\tilde{F}_+(k=0)$ or $\tilde{F}_+(k>0)$ and vice versa. This sounds difficult and cumbersome, however, in a typical regular EPG of periodic MRI sequences as shown in Fig. 1 it is only the transition from k=-1 to k=0 and vice versa.

(2) A more elegant and "fail-safe solution" is to work in only one domain, e.g. $\tilde{F}_+$. Then $\tilde{F}(0)$ is immediately well defined as $\tilde{F}_+(0)$ and any gradient transitions do not pose an issue. However, all $\tilde{F}_+(-k)$ states have to be complex conjugated to an $\tilde{F}_-(-k)$ state before the use in the RF pulse transfer matrix (Eq. 2), and all resulting $\tilde{F}_-(-k)$ states from the matrix calculation have to be converted back to an $\tilde{F}_+(-k)$ state by complex conjugation before storing it in the memory of the computer.

Summary

The EPG concept allows rapid and precise quantitation of magnetization response; however, care has to be taken of the complex spatial Fourier domain with its symmetry relations and the different effect of RF pulses and gradients on the transverse magnetization components. The author provides representative EPG software demonstrating the two approaches described in the solution section.