Distribution Specified Dipole Inversion for Quantitative Susceptibility Mapping

Yilin Yang[1], Tian Liu[2], Jianwu Dong[3], Pascal Spincemaille[4], and Yi Wang[4,5]

[1]*Department of Electronic Engineering, Tsinghua University, Beijing, Beijing, China,* [2]*MedImageMetric, LLC, New York, NY, United States,* [3]*Department of Automation, Tsinghua University, Beijing, Beijing, China,* [4]*Department of Radiology, Weill Medical College of Cornell University, New York, NY, United States,* [5]*Department of Biomedical Engineering, Cornell University, Ithaca, NY, United States*

Introduction: Dipole inversion is the final step of the quantitative susceptibility mapping (QSM) algorithm. In this step, the zero cone surface in the dipole kernel makes the field-to-susceptibility inverse problem ill-posed. Current solutions are mostly based on the Bayesian approach. Previous techniques have used weighted L1-norm to penalize the susceptibility gradient with binary weights derived from the gradient echo (GRE) magnitude image (MEDI [1]) or phase image (HEIDI [2]). Taking the information from the distribution of the susceptibility gradient into account could improve the reconstructed image. And L2-norm converges faster than L1-norm. Therefore, we employ reweighted L2-norm to specify the distribution to Gaussian, which is considered to be consistent with *a priori* knowledge. The results of this novel Distribution Specified Dipole Inversion (DSDI) method demonstrate an enhancement of QSM reconstruction and a significant shortening in calculation time.

Theory: Bayesian approach solves the susceptibility with the minimization form below.

$$\chi = \underset{\chi}{\operatorname{argmin}}\left[\lambda\|w(b-d*\chi)\|_2{}^2 + R(\chi)\right]$$

Here in the data fidelity term, χ is the susceptibility, d is the dipole kernel, b is the magnetic field and w is a weighting coefficient which compensate for the noise variation. The regularization term $R(\chi)$ contains the probability information of susceptibility, which is the main difference among current methods. According to Bayesian inference, the *a priori* probability is proportional to the exponent of the regularization term:

$$p(\chi) \propto e^{-R(\chi)}$$

Usually we use $R(\chi) = \|M\nabla\chi\|_p{}^p$ in a general term. Newton's iterative method could be used to solve this minimization problem with the $p \geq 1$, which is a convex problem.

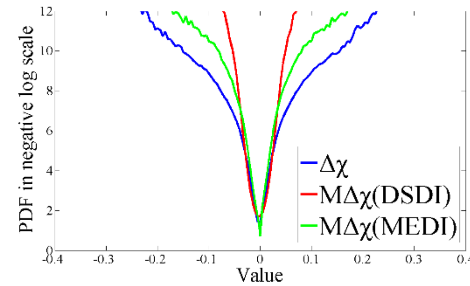

Figure 1: PDF of $M\nabla\chi$ in COSMOS with different weights

Calculation of susceptibility using multiple orientation sampling (COSMOS [3]) is regarded as ground truth and is used to analyze the distribution of $\nabla\chi$. The blue curve in Figure 1 is the negative log scale probability density function (PDF) of $\nabla\chi_{COSMOS}$. The distribution, however, is heavy-tailed indicating that the probability of high gradient is too large to accord with a convex problem. Although the weighted L1-norm with weights derived from GRE in MEDI [1] is applied, the negative log scale distribution, which is the green curve in Figure 1, is still unsatisfactory.

We aim to find weights M such that $M\nabla\chi$ obeys Gaussian distribution. If such M could be found, a regularization term using L2-norm, which converges the fastest, perfectly coincides the *a priori* assumption. By taking χ_{COSMOS} as training data and using a distribution specification to obtain the weights, the negative log scale distribution is converted into quadratic, indicated by the red curve in Figure 1. Although we could not obtain a perfect training data χ_{train}, we propose to implement reweighted L2-norm with weights renewed after each iteration, and use the susceptibility of the last iteration as training data. This novel reweighted L2-norm approach is called distribution specified dipole inversion (DSDI).

Methods: In the nth iteration step, we get the reconstructed image χ_n. The weights of the $(n+1)$th iteration step M_{n+1} are derived from χ_n and are used to solve χ_{n+1}. The method to train the weights with a training data consists of the following step: 1. Get the cumulative distribution function (CDF) of $|\nabla\chi_n|$ in terms of histograms. We divide the domain into k bins with width l, and the boundary of each bin is $a_i = il, i = 0,1,2,\ldots,k$. Define $CDF_{train}(i)$ as the proportion of points within $[0, a_i)$. 2. Choose a half Gaussian distribution with $\sigma = \sqrt{2\pi}\cdot\overline{|\nabla\chi_n|}$, so two distributions have the same mean value. The CDF of this half Gaussian distribution is $CDF_{Gauss}(x) = erf\left(\frac{x}{\sqrt{2}\sigma}\right)$. Here erf represents error function; 3. Convert the training data into the specified half Gaussian distribution. The modified boundary of each bin is $b_i = \sqrt{2}\sigma\, erfinv(CDF_{train}(i))$, where $erfinv$ is inverse error function. Transform all points in $[a_{i-1}, a_i)$ linearly into $[b_{i-1}, b_i)$ with $|\nabla\chi_{specified,i}| = (|\nabla\chi_{n,i}| - b_{i-1}) \times \frac{a_i - a_{i-1}}{b_i - b_{i-1}} + a_{i-1}$; 4. Calculate the weighting matrix with $M_{n+1} = \frac{|\nabla\chi_{specified}|}{|\nabla\chi_{train}|}$.

Results: The validation data are gadolinium phantom and the brain of one healthy volunteer. Image parameters were written in [4]. The magnetic field was calculated using the PDF [5] method. A result of the same brain using COSMOS [3] was provided as the ground truth. We examined DSDI as well as MEDI [1], and compared them with COSMOS both visually and statistically. Iterations of all methods cease when relative error is less than 0.1, which is considered convergent. Because the regularization parameter λ influences the image, a wide range of λ was evaluated and the best result was selected.

Figure 2 and Figure 3 display the in vivo and phantom QSM results with different methods. The coefficient of determination of linear regression (R^2) and calculation time are provided for both experiments. In the real brain experiment, the image of DSDI is in higher quality than MEDI. The susceptibility of basal ganglia region to which the arrows point is measured more precisely using DSDI. Moreover, DSDI results have a higher R^2 than MEDI and significantly abbreviate iteration time. In the phantom experiment, the two balloons at the top in MEDI have unpleasant streaking where the arrows indicate, but in both results of DSDI, the problem seems to be inconspicuous. A similar conclusion can be drawn from the phantom using the objective criteria that DSDI reconstructs QSM with higher R^2 while also reducing calculation time.

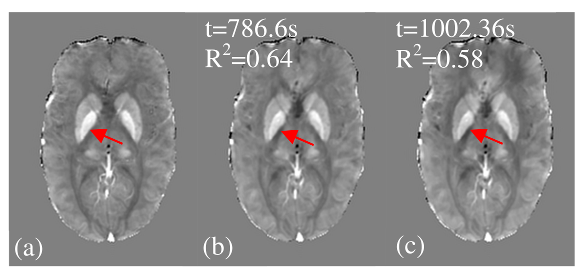

Figure 2: In vivo QSM of different methods: (a) COSMOS, (b) DSDI, (c) MEDI. Images of axial plane are shown.

Conclusion: In summary, DSDI uses reweighted L2-norm in dipole inversion. Weights are derived from the result of previous iteration to specify the distribution into Gaussian. DSDI improves reconstructed image quality and reduces iteration time. It is a promising method for quantitative susceptibility mapping.

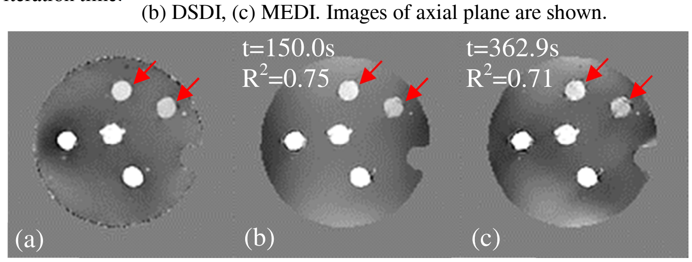

Figure 3: Phantom QSM of different methods: (a) COSMOS, (b) DSDI, (c) MEDI. Images of axial plane are shown.

References: [1] Liu J, et al. Neuroimage 2012;59(3):2560-2568. [2] Schweser F, et al. Neuroimage 2012;62(3):2083-2100. [3] Liu T, et al. Society of Magnetic Resonance in Medicine 2009;61(1):196-204. [4] Wang Y, et al. MRM-14-14978.R1. [5] Liu T, et al. NMR in biomedicine 2011;24(9):1129-1136.