regularized QSM with instant parameter sweep and reduced streaking artifacts in seconds

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<u>TARGET AUDIENCE</u>: Researchers interested in real-time Quantitative Susceptibility Mapping (QSM) reconstructions. <u>INTRODUCTION</u>: Fourier-based reconstruction of the underlying susceptibility distribution (χ) from the normalized local field map (φ) needs regularization to obtain a meaningful solution, and a sufficient spatial buffer around the region of interest (ROI) in the field of view (FOV) to avoid circular wrap-around. Recently a closed form deconvolution kernel was proposed that regularizes the gradient of the reconstructed χ in ℓ_2-norm[1], including a recipe to find the optimal balance between data-consistency and smoothness, represented with the parameter λ_{opt}. Together χ_{opt} and λ_{opt} can be used to initialize and accelerate more sophisticated ℓ_1-weighted and magnitude-based reconstruction algorithms[2]. <u>PROBLEM</u>: Although a single deconvolution now takes seconds, direct reconstruction is hampered as the optimal regularization weight can only be chosen based on a series of reconstructions with different regularization weights ("parameter sweeping"), making the total processing time an order of a magnitude higher. Another unresolved issue in QSM- reconstructions, is the optimal size of the spatial buffer to minimize wrap-around artifacts. <u>PURPOSE:</u> To obtain the optimal ℓ_2-regularization parameter within a second by minimizing the needed calculations, and to reduce wrap-around using low resolution aliasing removal[3].

<u>THEORY</u>: The closed form ℓ_2-solution minimizes the cost function $\left\|\mathcal{F}^{inv}D\mathcal{F}\chi-\varphi\right\|_2+\lambda\|G\chi\|_2$. The first term (data consistency) is the norm of the residual with D the dipole kernel and $\mathcal{F}$and $\mathcal{F}^{inv}$ the Fourier transforms. The second term is the multiplication of the regularization weight λ and the norm of the gradient of the reconstructed χ, with G the gradient operator. For a chosen λ, the closed form solution[1] is given by: $\tilde{\chi}_\lambda = D\left(D^2+\lambda\sum\tilde{G}^2\right)^{-1}\tilde{\varphi}$, where the tilde marks that the distribution is given in the Fourier domain. The optimal regularization parameter λ_{opt} can be found by plotting the regularization norm against the consistency norm for a series of reconstructions with varying values of λ. This yields a smooth L-curve in which λ_{opt} is associated with the point with maximal curvature[4].

<u>METHODS</u>: *<u>Sweeping</u>:* The consistency norm and the regularization norm were reformulated in the frequency domain (Parseval's theorem) using the discrete Laplacian filter $\widetilde{\nabla^2}=\sum\tilde{G}^2$. As the susceptibility map itself is not of interest during sweeping, the susceptibility in the norms was replaced with its closed form solution which can be further simplified to the formulas given in fig1, box2. The norms could be calculated over half Fourier space, using the conjugate symmetry of all matrices. *<u>Buffer handling</u>*: The field map φ_{in} was cropped leaving four voxels between ROI and FOV boundary, and then padded to create a cubic FOV. Residual wrap-around in the reconstruction was estimated and corrected for using an additional deconvolution of a down-sampled (½) model of the artificial periodic Fourier-environment of φ (fig1,box3). To compensate for scaling in this additional deconvolution the regularization weight of the Laplacian was reduced to ¼ λ_{opt}.

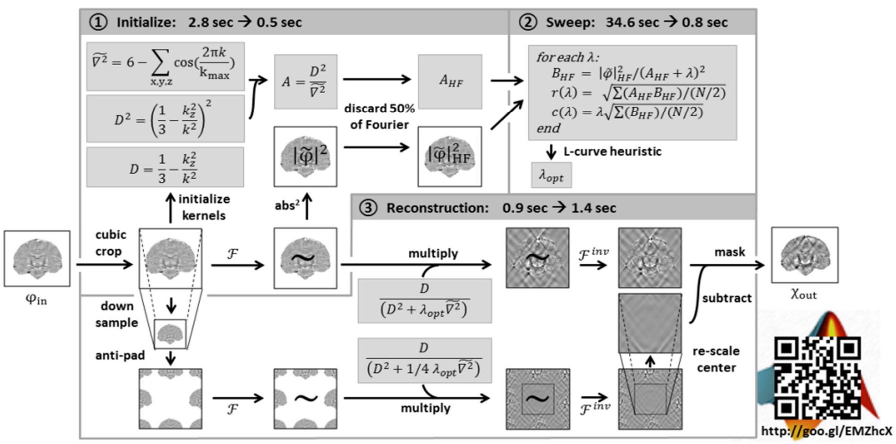

Fig1. Proposed scheme for ℓ_2-regularized QSM, with instant parameter sweep.
Box1: *The local field map φ_{in} was tightly cropped around the ROI to a cubic FOV and Fourier transformed. For the fast sweep only its power spectrum $|\tilde{\varphi}|^2_{HF}$ over Half Fourier domain (HF) was used, together with an auxiliary matrix A_{HF} , which was derived from the kernels for the closed form solution.*
Box2: *For each λ the regularization and consistency norm were calculated using only Half of Fourier domain, from a second auxiliary matrix B_{HF}*
Box3: *The susceptibility was calculated with the closed form solution, and residual wrap-around artifacts were estimated and removed with a 'virtual buffer', using an additional convolution over a down-sampled model of the periodic Fourier environment.*

<u>EXPERIMENTS</u>: *<u>Numerical</u>:* The performance of the virtual buffer was validated on a *masked* simulated field map (peakSNR = 100), which was calculated using a Fourier-based method[5] from a numerical χ-model[1]. For both the standard as the proposed method the regularization weight was set to $\lambda_{opt}=2*10^{-4}$. *<u>Experimental</u>:* The proposed sweep algorithm was tested on a publically available[2] *in vivo* acquired, masked normalized field map [3T,(0.6mm)^3,TE=8ms/20ms].

<u>RESULTS</u>: The anti-aliasing effect of a virtual buffer was practically equivalent to a conventional buffer for both the numerical (<0.005 ppm) and the experimental (<0.0017 ppm) dataset, while being four times faster. For the numerical experiment, the virtual buffer combined with a tight cubic FOV returned less low frequency streaking artifacts than the standard implementation, reducing the error of the χ reconstruction with 25%. Also in the experimental reconstructions a difference was observed regarding low frequency streaking (amplitudes upto 0.021 ppm, fig2c), which from visual inspection was believed to be an artifact from the standard implementation. The proposed sweep algorithm produced an equivalent L-curve, curvature plot and λ_{opt} (fig1a,b) for the experimental dataset, however reduced its calculation time about fortyfold. Combined, the total processing time of sweeping and reconstruction was reduced from 38.5 to 2.7 seconds.

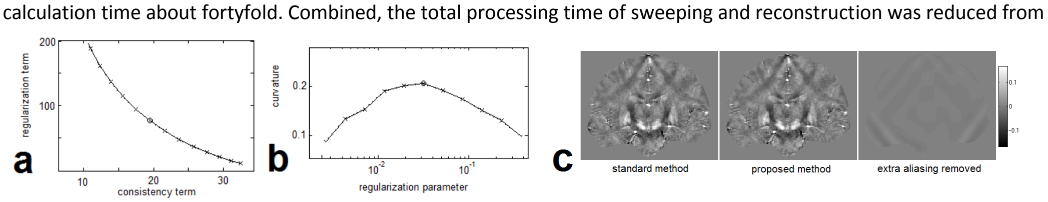

Fig2 *The L-curves **(a)** and curvature plots **(b)** of the standard (dashed) and the proposed sweep (continuous).* ***Fig2c*** *The susceptibility reconstructions and their difference.*

<u>DISCUSSION:</u> It was derived how the optimal ℓ_2-regularization parameter for QSM can be calculated in one second, which enables to include and optimize other regularization restrictions in the closed form solution. This fast sweep was presented as part of a framework that decouples aliasing prevention from the actual susceptibility reconstruction. Both the parameter sweep and the reconstruction can be performed using a tightly cropped cubic FOV. Although this cropping led to minor wrap-around streaks in the reconstructed susceptibility distribution, these Fourier-related artifacts were smooth due to the regularization, and could be removed with an additional deconvolution in a lower resolution. The fact that the optimal regularization parameter can be calculated from a cropped FOV, implies that voxels outside the ROI have a negligible contribution to the consistency and the regularization norm, an observation that may be valid and exploited in other, more computationally expensive QSM-reconstructions.

<u>CONCLUSION:</u> Optimally ℓ_2-regularized, aliasing-free QSM dipole inversion can be performed in seconds using an instant parameter sweep and a virtual buffer.

<u>REFERENCES</u>: **1.** Bilgic, JMRI, 40(1):181-91(2014) **2.** Bilgic, MRM, 72(5): 1444-1459(2014) **3.** Bouwman, MRM, 68(2):621-30 (2012) **4.** Hansen, SIAM J. Sci. Comput, 14(6):1487–1503(1993) **5.** Salomir, ConcInMagnReson, 19B: 26–34(2003) **<u>ACKNOWLEDGEMENT</u>**: B.Bilgic for sharing scripts and datasets.