# Multi-Echo Multi-Receiver MR Phase Reconstruction with Bipolar Acquisitions

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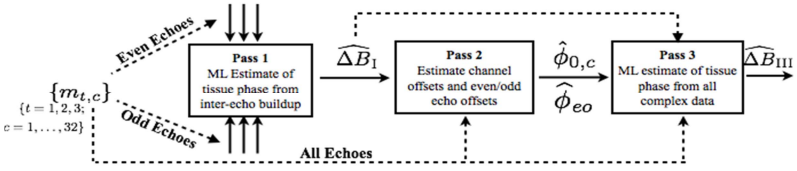
**Introduction**: Bipolar acquisitions of echoes in one TR allow for shorter inter-echo spacing and higher SNR efficiency than monopolar acquisitions, but introduce an unknown spatially varying phase offset between even and odd echoes. The processing and analysis of the resulting phase data across echoes, already facing ambiguities from phase wrapping, noise and channel-dependent phase offsets, is thus made even more challenging. Methods in the literature attempt to estimate this spatially varying even-odd echo offset, $\boldsymbol{\phi_{eo}(x,y)}$, but: (a) require additional reference scans, (b) neglect the offset along phase encode directions (c) make simplifying assumptions about spatial linearity of $\phi_{eo}(x,y)$ and/or (d) do not consider the general problem of imaging with an array of receive coils [1]. We propose here a strategy for reconstructing the underlying tissue phase from data acquired with an array of receive coils using bipolar echoes, without requiring a reference scan, and without making any assumptions about $\phi_{eo}(x,y)$. Our method separates the tissue phase, $\Delta B(x,y)$, from the phase offsets of the receive coils, $\boldsymbol{\phi_{0,c}(x,y)}$, and the phase offset due to even-odd echoes, $\phi_{eo}(x,y)$. **Methods**: The proposed phase reconstruction approach is based on a Maximum Likelihood (ML) framework [3] described using the 3-pass block diagram of Figure 1. *In the first pass*, raw measurements from all channels $c = 1,2,\ldots,N_c$ and echoes $k = 1,2,\ldots,NTE$ are separated into 2 groups: the even echo group and the odd echo group. We then pose the following MLE problem: what is the most likely tissue phase $\Delta B(x,y)$ that could explain the accumulated angle between even echoes only? The answer to this question does not require knowledge of $\phi_{eo}(x,y)$ nor $\phi_{0,c}(x,y)$: that is because we are only considering phase buildup within echoes of the same polarity. However, ambiguities due to phase wrapping and noise are present: we can show that this maximization problem will have multiple peaks (due to phase wrapping), each with potentially broad peaks (due to noise) [3]. We can pose a similar MLE problem for the group of odd echoes: this will yield another set of ambiguous solutions. We can show that the ambiguities from both even and odd echo groups could be removed by constraining the even and odd echo MLE problems to have a consistent solution (proof not included here). This is done formally in (1), where $L_{k1,k2;c}(\Delta B)$ is the likelihood of $\Delta B$ being the tissue phase that could explain the measurements from echoes $k_1$ and $k_2$, and channel c. We have derived this likelihood function in closed form, thus the solution to (1) is computationally efficient. **In the second pass**, we estimate $\phi_{eo}(x,y)$. To that end, we first remove the estimate of the tissue phase $\Delta B(x,y)$ obtained in Pass 1, from the angle of the original complex measurements, $m_{k,c}(x,y)$, as shown on right side of (2). Since the ML estimate is unbiased, any remaining ambiguity in the angle would be due to noise, $\phi_{eo}(x,y)$ and $\phi_{0,c}(x,y)$. A simple low pass filter operation on this resulting remainder, for every echo and every channel, would then recover a quantity proportional to $R_e = \exp(i(\phi_e(x,y) + \phi_{0,c}(x,y)))$, for even echoes, and $R_o = \exp(i(\phi_o(x,y) + \phi_{0,c}(x,y)))$, for odd echoes. Dividing $R_e$ by $R_o$, and summing over all channels, yields an estimate of $\exp(i(\phi_{eo}(x,y))) = \exp(i(\phi_e(x,y) - \phi_o(x,y))$. **In the third pass**, we refine the tissue phase estimate obtained in Pass 1. To do this, we multiply the original data $m_{k,c}(x,y)$ by the conjugate of $R_e$ (for even echoes) and $R_o$ (for odd echoes), thereby removing both the channels' phase offset and the even-odd echo offset from all complex measurements. We can thus pose the following ML problem: what is the most likely tissue phase $\Delta B(x,y)$ which can explain all measurements?




$$\widehat{\Delta B}_{\mathrm{I}} = \arg\max_{\Delta B} \prod_{\mathrm{k1,k2}\,=\,1,2,..,\mathrm{NTE}} \prod_{c} L_{k1-k2,c}(\Delta B) \qquad (1)$$

$$\mathrm{mod(k1,2)} = \mathrm{mod(k2,2)}$$

$$(\phi_{0,c} + \phi^{\mathrm{mod(k,2)}}) = \angle\left\{\sum_{k} \mathcal{LPF}\{m_{k,c}\exp(-i2\pi\widehat{\Delta B}_{\mathrm{I}}\mathrm{TE}_k)\}\right\} \qquad (2)$$

$$\widehat{\Delta B}_{\mathrm{III}} = \arg\max_{\Delta B} \prod_{k=1}^{\mathrm{NTE}} \prod_{c} L'_{k,c}(\Delta B) \qquad (3)$$

**Figure 1 –** Block diagram of the proposed ML estimator from bipolar acquisitions

We formulate this problem in closed form (3). Similar to the first pass, the maximization is performed voxel-by-voxel, and thus could be rapidly performed using brute-force search methods. We make this final important note: for arbitrary choices of echo times, the solutions to Eqs. (1) and (3) may not have a single and/or sharp maximum. To address this issue, we have made use of a previously developed method, MAGPI in [1,3], to optimize NTE (NTE>2) echoes at which these ML problems have the smallest MVU estimate possible. Details about the optimizer are beyond our scope here. **Acquisition**: We collected two 3D MEGE scans on a 3T Siemens Skyra with a 32-channel heal coil: the first scan used monopolar acquisitions (TR=41ms), while the second scan used bipolar acquisitions (TR=34ms), both with NTE=5 echoes. The following were common scan parameters: 3D FOV 220(r.o.)x206mm with 48 slices of thickness 2mm, flow compensation along read-out, matrix size 256x240. The minimum echo time spacing $\Delta$TEmin achievable with monopolar readout was 5.86ms whereas $\Delta$TEmin of bipolar echoes was 4.31ms. The monopolar echo times were TE = 9.67, 16.64, 22.50, 28.36, 34.23ms, whereas the bipolar echoes were: 7.44, 12.3, 16.75, 23.62, 29.37ms. The echoes were chosen by the MAGPI optimizer of [3] for optimal ML phase reconstruction. **Results:** Figures 2a and 2b show the tissue phase $\Delta B(x,y)$ obtained with the monopolar and bipolar echoes, respectively. Figure 2c shows example estimates of the even-odd echo bipolar phase offset, $\phi_{eo}(x,y)$, obtained before averaging $R_e/R_o$ over the 32 channels (see Pass 2). Figure 2d shows the resulting $\phi_{eo}(x,y)$ after averaging over coils. **Discussion:** The following two observations indicate the accuracy of our phase estimations with the bipolar acquisition. First, we note the close similarity between the $\Delta B$ estimates obtained with the monopolar acquisition (Fig 2a) and that obtained with bipolar acquisition (Fig 2b). Second, the estimates of $R_e/R_o$ obtained at each channel are almost identical (Fig 2c): this is expected since $\phi_{eo}(x,y)$ does not change through the channels. This cross-channel consistency was not directly imposed in reconstruction, thereby highlighting the robustness of our framework. Finally, we note from Figs 2d and 2f that our method is able to recover arbitrary forms of $\phi_{eo}(x,y)$, which display here a non-linear spatial behavior.

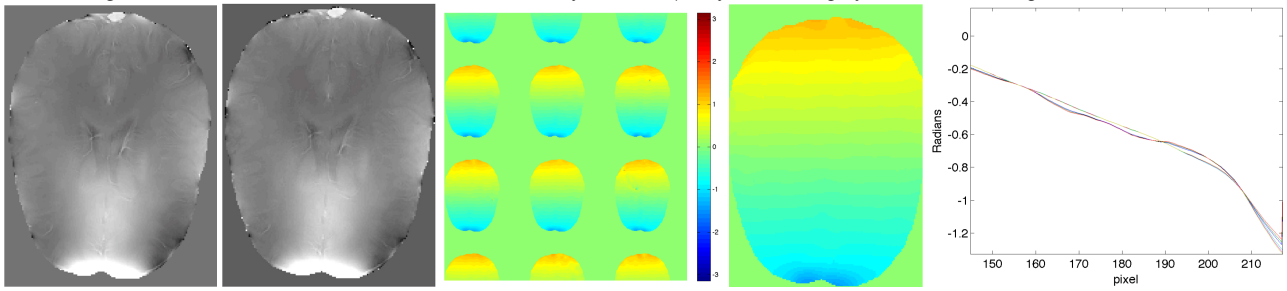



**Figure 2 –** Estimate of the tissue phase using (a) monopolar and (b) bipolar acquisition. (c) For the bipolar acquisition, an estimate of the even-odd echo offset $R_e/R_o$ obtained for each channel independently, then combined in (d). (f) Profile through the combined $\phi_{eo}$ showing the non-linear profile across the read-out lines

**References**: **[1]** Yu H. *et. al.,* JMRI 2010; 5:1264-71. **[2]** Dagher J, Reese T, Bilgin A, MRM 2013; 71:105-17. [3] Dagher J *et. al*, ISMRM 2014, 3263.