PDE Solution of Electrical Properties Tomography With Multi-channel B1 Transmission

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Audience Researchers or physicians interested in topics including: EPT, MRI Safety, MRI RF Engineering, Electromagnetic Modeling, etc.

Purpose Electrical properties tomography (EPT) is a recently introduced technique for imaging the electrical conductivity and permittivity of tissue based on the measured radiofrequency (RF) field (B_1) distribution in MRI [1,2]. *In vivo* electrical properties (EP) can potentially benefit clinical diagnosis of diseases and RF heating quantification associated in high-field MRI applications [3]. The EPT equation can be transformed into a partial different equation (PDE) and solved using numerical PDE schemes such as finite element [4] or finite difference method [5]. The advantage of the PDE-based approach consists of improved EP result near the boundary and enhanced robustness against noise contamination. In this study, a multi-channel transceiver RF coil was utilized to provide multiple transmit B_1 field for solving the PDE. Differing from previous studies [4,6], the method here does not require assumption of equal transmit and receive RF phase which may be violated at higher field, and the need of pre-assigned boundary condition [4] was eliminated by using multiple excitations, beneficial for *in vivo* applications.

Theory By taking $\zeta = 1/\varepsilon_c$ and ignoring the z-component of B_1, the relationship between transmit B_1 field (B_1^+) and ζ is described in eq. (1), where

$$\left(\frac{\partial B_1^+}{\partial x} - i\frac{\partial B_1^+}{\partial y}\right)\left(\frac{\partial \zeta}{\partial x} + i\frac{\partial \zeta}{\partial y}\right) + \frac{\partial B_1^+}{\partial z}\frac{\partial \zeta}{\partial z} + \nabla^2 B_1^+ \zeta \approx -\omega^2 \mu_0 B_1^+ \quad (1)$$

$\varepsilon_c = \varepsilon - i\sigma/\omega$, σ is the conductivity, ε the permittivity, ω the angular frequency and μ_0 the permeability of free space. It is noticed that with at least three independent excitations, the three items of $\partial\zeta/\partial x + i\partial\zeta/\partial y$, $\partial\zeta/\partial z$ and ζ can be determined voxel-wisely through solving a group of linear equations, and a unique EP solution is derived without an additional boundary condition. Similarly, when EP is assumed to be constant along the z-direction, two independent excitations are needed for determining a unique EP distribution in a slice.

Methods ***Simulation*** To investigate the feasibility of solving eq. (1) using multiple B_1^+ without a boundary condition, simulated B_1^+ was generated using finite-difference time-domain method. A 16-channel stripline RF coil [7] as shown in Fig. 1(a) was modeled in software SEMCAD and loaded with a realistic human head model (Ella) as shown in Fig. 1(b). The simulation grid size was set to 2x2x2 mm^3. A total of sixteen complex B_1^+ at 298 MHz were simulated for all coil elements. Assuming $\partial\zeta/\partial z = 0$, the PDE of eq. (1) was solved using a 2D finite difference method [5]. The method was tested with different number of transmit channels, i.e., 2 (#1 and 9), 4 (#1, 5, 9 and 13) and all 16 channels were chosen to provide complex B_1^+, respectively. ***Experiment*** MRI experiment on a three-compartment gel phantom was conducted on a 7T MRI scanner (Siemens, Erlangen, Germany) equipped with the 16-channel stripline RF coil in Fig. 1(a), powered by 16x1kW RF amplifiers. EP of the phantom was measured using a dielectric probe (85070D, Agilent, Santa Clara, CA, USA). Data including magnitude $|B_1^+|$ and *relative* phase $\angle B_{1r}^+$ of all sixteen transmit channels were acquired using a hybrid B_1-mapping technique [8,9]. In order to derive the *absolute* phase $\angle B_1^+$, eq. (1) was first solved using magnitude $|B_1^+|$ and *relative* phase $\angle B_{1r}^+$[10]. Once $\angle B_1^+$ is obtained, eq. (1) was solved with the 2D finite different method using complex transmit B1 of all sixteen channels.

Results ***Simulation*** Fig. 1 (c-j) summarize the results of reconstructed EP using different number of transmit channels. As we can see, two channels are sufficient to derive a unique solution without using any boundary condition, while residual artifacts within certain regions in the 2- and 4-channel setup can be observed in Fig. 1(e-h). This is due to incomplete coverage of the whole slice with limited number of channels. Hafalir et al. [4] indicated that regions with weak $\partial B_1^+/\partial x - i\partial B_1^+/\partial x$ introduce more severe numerical error, which was largely mitigated as all sixteen channels were utilized as shown in Fig. 1 (i, j). The relative error (RE) and correlation coefficient (CC) of the reconstruction results using 2, 4 and 16 channels are $RE_{\sigma,2}$=19.6% / $RE_{\varepsilon,2}$=14.1% / $CC_{\sigma,2}$=0.16 / $CC_{\varepsilon,2}$=0.02, $RE_{\sigma,4}$=15.0% / $RE_{\varepsilon,4}$=10.8% / $CC_{\sigma,4}$=0.68 / $CC_{\varepsilon,4}$=0.39, $RE_{\sigma,16}$=7.7% / $RE_{\varepsilon,16}$=9.1% / $CC_{\sigma,16}$=0.93 / $CC_{\varepsilon,16}$=0.73, respectively. ***Experiment*** With the measured $|B_1^+|$ as shown in Fig. 2(a,c) and $\angle B_{1r}^+$ as shown in Fig. 2(b,d), calculated gradients of the absolute phase $\angle B_1^+$ of channel #16 are illustrated in Fig. 2(e) and (f), respectively. The center of the images were masked out due to extreme low B_1 resulted from the employed B1 shim setting (CP2+ like [11]). As we can see in Fig. 2 (h), the reconstructed σ distribution clearly delineates the boundary of components and the σ value agrees with the probe measurement as shown in Fig. 2 (g), with reconstructed σ_1=0.61±0.08 Sm^{-1}, σ_2=1.35±0.26 Sm^{-1} and σ_3=0.47±0.19 Sm^{-1}, in comparison with probe-measured values of 0.84, 1.60 and 0.56 Sm^{-1}, respectively.

Conclusion & Discussion In this study, a PDE-based approach to solve EPT problem using multi-channel transmission was investigated. A PDE-based approach is merited with improved performance near boundary and less severe artifact due to noise contamination. In the phantom experiment, it is shown that with 16 transmit channels, a unique EP solution can be found in consistent with the ground truth without using boundary conditions which are possibly not accessible under *in vivo* situations. The retrieved phase in Fig. 2(e, f) and reconstructed conductivity in Fig. 2(h) hold significant values for clinical diagnosis and RF safety applications.

References [1] Wen, *Proc. SPIE* 2003, 5030:471;[2] Katscher et al., *IEEE TMI* 2009, 28:1365; [3] Voigt et al., *MRM*, 2011, 66:456; [4] Hafalir et al., *IEEE TMI*, 2014, 33:777; [5] Iserles 2009; [6] Gurler et al., *ISMRM* 2014; [7] Adriany et al. *MRM* 2008, 59:590; [8] Van de Moortele et al., *ISMRM* 2007, 1676; [9] Van de Moortele et al., *ISMRM* 2009; [10] Liu et al., *MRM*, 2014 online; [11] Orzada et al., MRM, 2013.

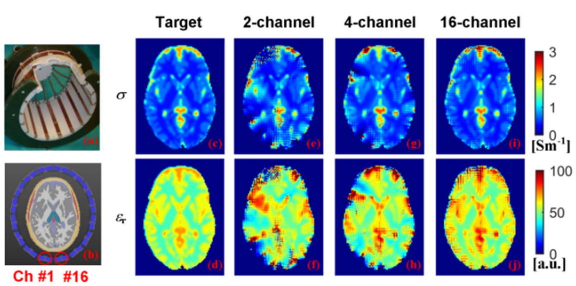

Fig. 1 Result in stimulation study. (a) The physical 16-channel coil utilized in the experiment. (b) Model of the coil loaded with Ella head model. (c, d) Target electrical properties. (e-j) Results of reconstructed EP using different number of channels.

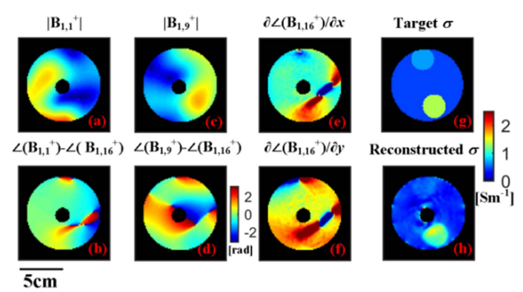

Fig. 2 Result in experiment study. (a, c) Measured magnitude $|B_1^+|$ of channel #1 and #9, respectively. (b, d) Measured relative phase $\angle B_{1r}^+$ of channel #1 and #9 referring to channel #16, respectively. (e,f) Calculated derivatives of $\angle B_1^+$ of channel #16 along x- and y-direction, respectively. (g) Probe-measured conductivity of the phantom. (h) Reconstructed conductivity.

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