Low pass filter based electrical property tomography (EPT) reconstruction

Jaewook Shin[1], Min-oh Kim[1], Narae Choi[1], and Dong-Hyun Kim[1]
[1]Electrical and Electronic Engineering, Yonsei University, Seodaemun-gu, Seoul, Korea

Target audience: Researchers with interests in electrical property mapping.

***Purpose*:** Magnetic resonance electrical properties tomography (MREPT) is currently being investigated for many clinical applications (1,2). However, MREPT suffers from statistical noise and boundary artifact (3). Especially, the noise amplification in MREPT is occurred due to the calculation of the Laplacian operator. To overcome this EPT error, filtering or fitting based technique was introduced (2). In this study, low pass filter (LPF) based EPT reconstruction method without the Laplacian operator is proposed.

Theory: For conventional MREPT, the admittivity ($\gamma = i\omega\mu_0(\sigma + i\omega\varepsilon)$) information is retrieved using Eq. 1 (H: magnetic field). In 2D image domain, a simple discrete Laplacian operator (L_d) can be defined as a convolution kernel and it can be decomposed into two components due to its linearity (Eq. 2). The frequency responses of L_d and the first component (L_m, C=4) are presented as Fig. 1a,b. This first term (L_m) can be designed as a conventional LPF (Fig. 1c) by linearly combining the other L_d (4) and choosing appropriate *C* value. From this, the conductivity (σ) and permittivity (ε) information can be extracted as Eq. 3 by substituting Ld for LPF.

$$\gamma(\mathbf{r}) = L_d(\mathbf{r}) * H(\mathbf{r})/H(\mathbf{r}) \quad (1)$$

$$L_d(\mathbf{r}) = \begin{bmatrix} & 1 & \\ 1 & -4 & 1 \\ & 1 & \end{bmatrix} = \begin{bmatrix} & 1 & \\ 1 & C & 1 \\ & 1 & \end{bmatrix} + \begin{bmatrix} & 0 & \\ 0 & -C-4 & 0 \\ & 0 & \end{bmatrix} = L_m(\mathbf{r}) - (C+4)\delta(\mathbf{r}) \quad (2)$$

$$\sigma(\mathbf{r}) = \mathrm{Re}\left\{\frac{LPF_1(\mathbf{r}) * H(\mathbf{r})}{i\omega\mu_0 LPF_2(\mathbf{r}) * H(\mathbf{r})}\right\}, \varepsilon(\mathbf{r}) = \mathrm{Im}\left\{\frac{LPF_1(\mathbf{r}) * H(\mathbf{r})}{i\omega\mu_0 LPF_2(\mathbf{r}) * H(\mathbf{r})}\right\} - \frac{C+4}{i\omega\mu_0} \quad (3)$$

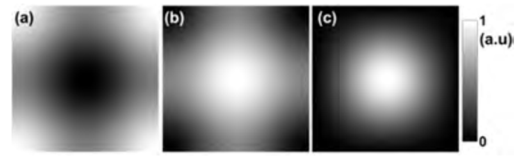

Fig 1 Normalized frequency response of (a) simple discrete Laplacian operator, (b) the first component (L_m, C=4) in Eq. 2 and (c) modified kernel by linearly combining different L_{DLO}.

Method: For LPF based reconstruction, Gaussian filter (LPF_1) was employed to calculate the discrete Laplacian operator. For additional denosing, the denominator term in Eq. 3 is smoothed by using Gaussian filter with a relatively smaller kernel (LPF_2). To compare with conventional method, Gaussian filtering with various kernel sizes was employed after calculating discrete Laplacian operator. Conductivity error was evaluated with root-mean-square error (RMSE) over homogeneous region. For experiment, a cylindrical phantom with a diameter of 15mm was filled with a mixture of 1.0% agar and 0.5% NaCl. Experiment was performed in a 3T clinical scanner (Siemens Tim Trio) using 3D TrueFISP (α=30°, TR/TE≈4.8 /2.4ms with 4 average, voxel size=1×1×1 mm^3) for phantom and Spin Echo (TR/TE=1000/12ms, voxel size=2×2×4 mm^3) for in-vivo brain. Conductivity map was reconstructed using only B_1 phase (5).

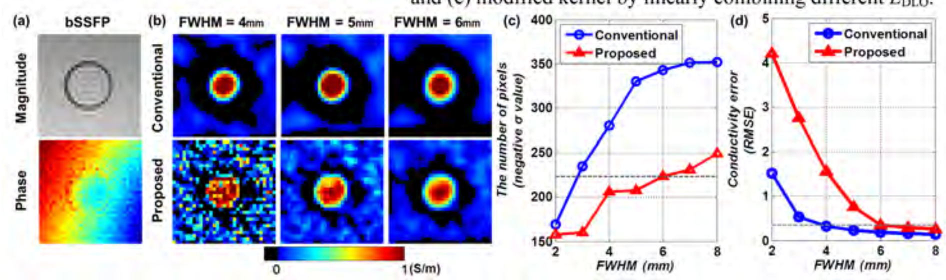

Fig 2 (a) balanced SSFP magnitude and phase. (b) Conductivity map reconstructed using conventional and proposed method. For proposed method, all kernel size of LPF_2 was fixed to 1.0 mm (In conductivity map, the region with black color indicates negative conductivity value. FWHM: Full-Width at Half Maximum). (c) The number of pixels with negative conductivity value (i.e. boundary artifact) and (d) conductivity error (RMSE) as a function of kernel size.

Result & Conclusion: In Fig. 2b and 2d, for small size kernel (FWHM≤4 mm), conductivity results of proposed method noisier than the results of conventional method. However, as kernel size increases (FWHM=5,6 mm), the proposed method effectively reduces the noise in conductivity map and simultaneously prevented boundary artifact (negative σ value, black region in Fig. 2b) from spreading to adjacent regions. The number of pixels with negative conductivity value shows that the proposed method is less sensitive to boundary artifact broadening (Fig. 2c). Similar results were observed for in-vivo brain conductivity map (Fig. 3). Especially, at the tissue boundaries, the proposed method preserves the conductivity values. Hence, the proposed method can give boundary artifact minimized conductivity map by flexibly choosing kernel size of $LPF_{1,2}$. This trade-offs of filter size can be easily determined since the operation does not involve a Laplacian operator in the original admittivity equation (Eq. 1). The method seems to work better for minimum kernel size of LPF_2 and also for high SNR data which is essential for EPT. In practice, a scaling factor should be compensated after low pass filtering, because the LPFs cannot be substituted exact a Laplacian operator. This scaling factor depends on the difference between FWHM of LPF_1 and LPF_2.

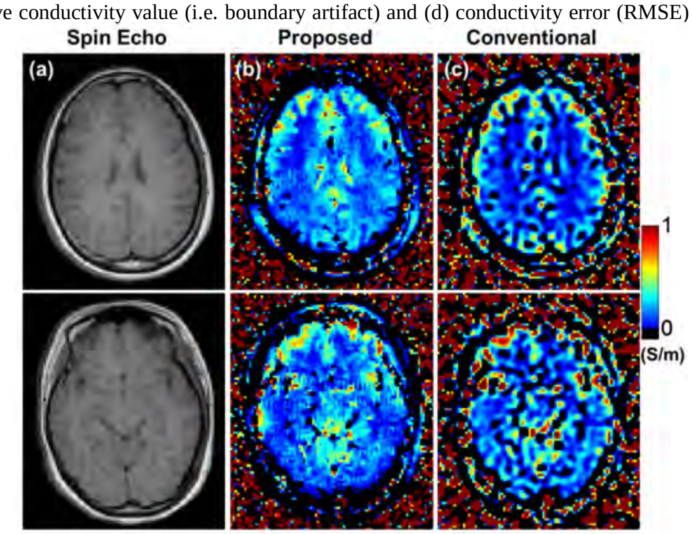

Fig 3 (a) Spin Echo image and Conductivity map reconstructed by using (b) proposed method and (c) conventional method. For conventional method, Gaussian filtering with FWHM = 3.0mm was applied. For proposed method, FWHM of $LPF_{1,2}$ were 8.0 and 1.0 mm. (In conductivity map, the region with black color indicates negative conductivity value.)

References: 1. E Balidemaj et al, MRM (2014) 2. U Katscher et al, ISMRM 21 (2013) 3372 3. JK Seo et al, IEEE TMI (2012) 430-437 4. Lindenberg, T PAMI(12), No. 3, March 1990, pp. 234-254 5. Voigt et al, MRM(2011) 66:456–466 ***Acknowledgements:*** NRF grant funded by the Korea government (MEST) (No. 2012-009903)