A Regularized Model-Based Approach to Phase-Based Conductivity Mapping

Kathleen M Ropella[1] and Douglas C Noll[1]
[1]*Biomedical Engineering, University of Michigan, Ann Arbor, MI, United States*

Target Audience: MR scientists and clinicians interested in conductivity mapping using MRI.

Purpose: Current approaches to conductivity mapping in vivo using MRI primarily involve convolving a Laplacian kernel with acquired transmit RF field data and using spatial filtering techniques to reduce noise levels. This work describes a regularized, model-based approach to conductivity mapping, which is more robust in the presence of noisy phase maps and yields more accurate estimates near boundaries.

Methods: In 2003, Wen[1] showed that the phase of the B_1^+ field is primarily affected by the conductivity of an object, giving rise to phase-based conductivity mapping (Eq. 1). Instead of solving the forward problem to determine conductivity from measured phase by convolving with a Laplacian kernel (Eq. 2), we solve the inverse problem by finding the conductivity map most likely to produce the measured phase map, in a least-squares sense. The cost function is given in Eq. 3, where σ is the conductivity map, φ^+ is the measured B_1^+ phase, *A* is the inverse of the Laplacian operator - calculated by inverting the DFT coefficients of the kernel given in Eq. 2 - β is the regularization parameter, and the regularization function *R* is an edge-preserving roughness penalty. The weighting function W_1 is 1 within the object and 0 elsewhere and W_2 is 0 at edges and 1 elsewhere.

A numerical phantom was created using two cylinders with conductivities of 0.587 S/m and 2.143 S/m. A quadrature birdcage coil was also simulated at 128MHz. Complex AWGN was added to simulated B_1^+ fields prior to reconstruction. The SNR of the simulated B_1^+ maps was varied to compare the RMSE and SNR of the reconstructed conductivity maps in the presence of different levels of noise in the input data. SNR was calculated as 20*log(mean/standard deviation) in the inner compartment. Human subject data was acquired using a GE Discovery MR750 3.0T scanner (GE Healthcare, Waukesha, WI) and a spin echo sequence with TE/TR = 16/1000ms, FOV = 24x24cm, 1.25x1.25x3mm voxels. Conductivity maps were reconstructed using the proposed method, hereafter called the Inverse Laplacian method, and a basic filtering method, which included applying a 5x5 Gaussian filter with a standard deviation of 2 pixels before and after convolving phase data with the Laplacian kernel. Simulations were performed using SEMCAD X[2]. Image processing and model-based reconstruction was performed using MATLAB (The Mathworks, Natick, MA) in conjunction with J. Fessler's Image Reconstruction Toolbox[3].

Eq. 1: $\sigma \approx \dfrac{\nabla^2\varphi^+}{\omega\mu_0}$ **Eq. 2:** $[\nabla^2] = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ **Eq. 3:** $\hat{\sigma} = \operatorname{argmin}_\sigma \frac{1}{2}\|\varphi^+ - A\sigma\|_{W_1}^2 + \beta W_2 R(\sigma)$

Results: Simulations show lower RMSE and higher SNR for conductivity maps when using the Inverse Laplacian method over a wide range of B_1^+ map SNR values. In the region where spatially filtering conductivity maps yields higher SNR values, RMSE suffers from boundary artifacts and negative conductivity values. In the human brain, average conductivity values for gray matter, white matter, and CSF were 0.94, 0.45, and 1.19 S/m, respectively, when reconstructed using the Inverse Laplacian method. Reported values are 0.59, 0.34, and 2.14 S/m, respectively[4].

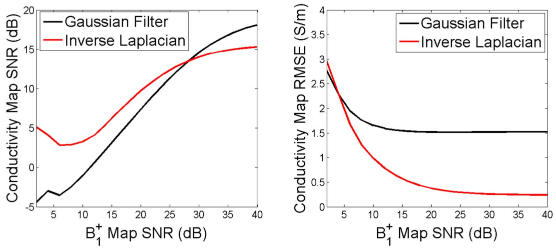

Figure 1: Comparison of conductivity map SNR (left) and RMSE (right) as a function of B_1^+ map SNR for both reconstruction methods using simulation data.

Discussion: The Inverse Laplacian method provides conductivity map reconstructions with more accurate conductivity values and limited boundary artifact while providing adequate SNR to distinguish anatomical features. When using spatial filtering methods, boundary artifacts are prominent in the human brain data along with artifacts near phase discontinuities at major vessels. Additionally, the Inverse Laplacian method produces only non-negative conductivity values whereas spatial filtering results in some negative-valued regions.

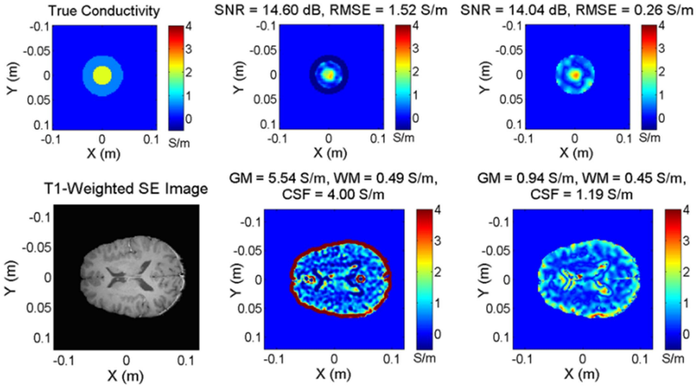

Figure 2: Simulation with B_1^+ map SNR = 30dB (top row) and human brain (bottom row) results. Reference images (left) and reconstructed conductivity maps using Gaussian filters (center) and the Inverse Laplacian method with β=2^4 (right).

References: [1] H. Wen, SPIE, vol. 5030, 2003. [2] SEMCAD X by SPEAG, www.speag.com. [3] J. A. Fessler et al., IRT. http://web.eecs.umich.edu/~fessler/code/index.html. [4] P. A. Hasgall, et al., "IT'IS Database," Version 2.5, 2014.