Simulating Charge at Electrical Property Interfaces

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Introduction: Conductive materials present both challenges and opportunities in MRI. Metal prosthetics create field inhomogeneities that can degrade image quality. Conversely, electrical property maps (EPMs) give important structural and physiological information[1]. Our group is particularly interested in developing methods to measure low-frequency EPs non-invasively. Recently, novel pulse sequences, such as the one shown in Fig. 1, have been proposed that use eddy-currents induced by time-varying gradients to measure low-frequency EPs[2]. It has been suggested that charge accumulates at EP boundaries and this charge will impact the MR signal[3]. This abstract proposes and demonstrates a computational algorithm that accounts for induced charge and tracks the resulting electromagnetic fields during a rising gradient pulse.

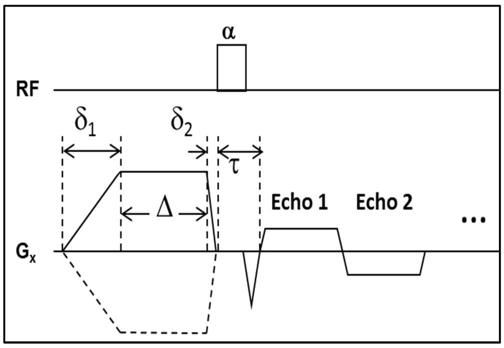

Fig. 1: Proposed sequence that uses rapidly varying gradients prior to excitation to induce eddy-currents that can be used to map EPs at low frequencies non-invasively.[2]

$$\frac{\partial B}{\partial t} = -\nabla \times E_{curl} \quad (1)$$

$$\epsilon\mu\frac{\partial E_{full}}{\partial t} = \nabla \times B - \mu\sigma E_{full} \quad (2)$$

$$\frac{\partial \rho_{free}}{\partial t} = -\frac{\sigma}{\epsilon}\rho_{free} - E \cdot (\nabla\sigma - \sigma\nabla\ln(\epsilon)) \quad (3)$$

$$U_D = \frac{1}{4\pi}\int \frac{\rho_{free}\left(r', t - \frac{r-r'}{c}\right)}{(r-r')} dV' \quad (4)$$

$$\frac{\nabla \cdot U_D}{\epsilon} = E_\rho \qquad E_{full} = E_\rho + E_{curl} \quad (5)$$

Mathematical Theory: In divergence-free fields, Faraday's law and Ampere's law, Eqs. (1) and (2), are sufficient to calculate the electromagnetic fields. In general however, these equations do not account for charge contributions to E nor predict how charge accumulates. To understand the charge dynamics, one examines the current continuity equation under the assumption of Ohm's law. Assuming a linear material, $\nabla \cdot \left(\frac{\sigma D}{\epsilon}\right) = -\frac{\partial \rho_{free}}{\partial t}$, where ρ_{free} is the free charge density. Expanding and using $\nabla \cdot D = \rho_{free}$ gives the differential equation shown in Eq. (3). Note this equation predicts free charge will only accumulate in areas with discontinuous EPs. Solving for ρ_{free}, one applies Helmholtz theorem with a time-retarded charge source as shown in Eq. (4), where r' is the source location and the integral is over the entire charge density. Finally the charge contributions to E are given by the divergence of the potential as shown in Eq. (5).

Simulation Algorithm: We simulated the effects of eddy-currents induced by gradient switching in tissue. The algorithm consists of two parts. The first uses finite-difference forms of Faraday and Ampere's law along with Ohm's law to iteratively solve for E and B. This algorithm is standard and well-outlined by Zhao and Turner[4]. The simulated phantom is a rectangular prism with dimensions 8x8x8 cm. σ and ϵ increase smoothly in the interior of the phantom peaking with values of 1.2 S/m and 80 ϵ_0 at the center approximating physiological values. dt and dx_i of the simulation are bound by the equation $\frac{dx_i}{dt} \approx c$. For a given FOV, increased resolution comes a double cost as the matrix operations at each time step scale linearly with grid size, and the amount of progress in time for each step is reduced as well. All considered, the resolution of individual field components was chosen to be isotropic 1 cm and dt to be 5.77 ps, comparable to previous MRI FTDT simulations[5]. A linear gradient in z is set up by two Helmholtz coils above and below the phantom.

The second portion of the algorithm uses E calculated from the first portion to determine the discrete charge distribution using the finite-difference equivalent of Eq. (3). Noting that Helmholtz theorem is equivalent to $U_D = \frac{1}{r} * \rho_{free}$, one can solve for U_D quickly using the convolution theorem and the FFT. At ps temporal resolution, the potential has to be treated as time-retarded. Once the potential is determined its gradient can be used to correct E. The process is repeated iteratively.

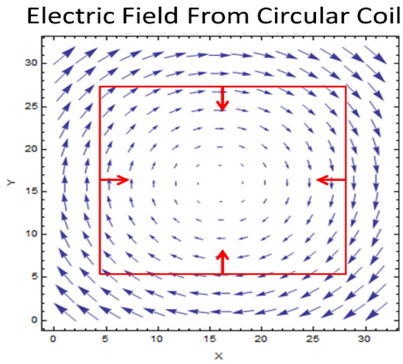

Fig. 2: A conceptual vector field map of the E-field induced by a rising current in a coil. Charge initially accumulates where E and the surface norms are most parallel/anti-parallel at the corners.

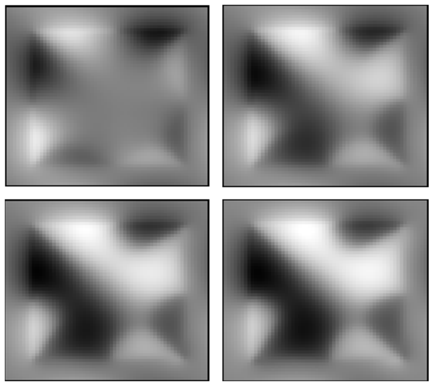

Figure 3: The simulated voltage due to accumlated charge on the top surface of an 8x8x8 cm phantom in a rising gradient field at about 40, 80, 120, 160 ns (right to left, top to bottom) after the pulse start.

Discussion and Conclusions: Distinct patterns of charge accumulation appear soon after the current is applied. Eq. (3) indicates charge accumulates most rapidly at locations where E has a component parallel or anti-parallel to the phantom's surface norms. Initially when no charge is present, E is generated only by the coil. Fig. 2 shows E resulting from increasing current in a coil with no load. The red box outlines the phantom and shows locations where we can conceptually evaluate potential charge distributions. In this simplified arrangement, one expects charge to accumulate most rapidly in the corners. The growing charge changes E but the simulation establishes a distinct charge pattern that grows in magnitude over time. The voltage resulting from accumulated charge in our simulation at different times is shown in Fig. 3. While charge responds to a fixed E with a characteristic time of $\frac{\epsilon}{\sigma}$, roughly $10^{-10}\,s$, feedback between ρ and E cause both to change continuously through the pulse rise and perhaps beyond. Early in the pulse rise time, the linear gradient inside the material is not greatly impacted, but the fields outside the phantom are influenced by transients. In conclusion, we developed an algorithm to evaluate the effect of charge accumulation at boundaries due to induced currents during time-varying gradients. This method may give insight into correcting image artifacts caused by conductive materials and guide efforts to map low-frequency tissue electric properties non-invasively.

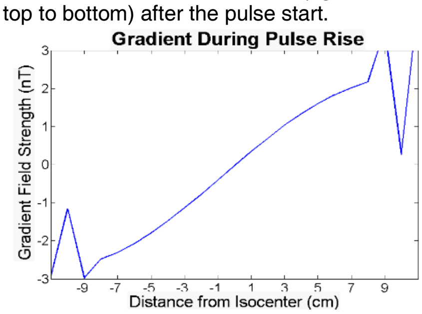

Figure 4: Gradient at about 160 ns after pulse start.

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