

# Stability Test Method for Cartesian Feedback Power Amplifier in pTx Array

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**Introduction:** A 7 Tesla pTx system under construction at ELH Institute for Magnetic Resonance Imaging, Essen, Germany will employ 32 near-magnet power amplifiers (PA) of 1kW output power [1]. As magnetic circulators / isolators have to be avoided near the MR magnet, close “tuned” connection of PA output and coil causes mismatch of the PA due to load variation and coupling from neighbour channels; Fig.1 shows the principle configuration of the PAs feeding an array of coupled Tx coils. Variation of coil current due to load variation can be compensated by a Cartesian feedback loop implemented in the PA which senses the output voltage as explained also in [1]. However, variations of load impedance may also lead to instability of the feedback loop. In order to avoid instability in the pTx system, conditions and limits of stability have to be investigated for every possible mode of operation. In this contribution, we present the principle architecture of our PA with control loop and propose an efficient method of stability check which allows extension of stability investigations to the large pTx array.

**Concept of PA:** The architecture of the Cartesian feedback power amplifier is sketched in Fig.2. The output voltage  $V_{out}$  of the final stage power amplifier is sampled by a voltage divider and is compared to the RF input signal  $V_{in}$  from the system RF exciter. The difference signal is I/Q down-converted to baseband and low-pass filtered to control a modulator (up-converter) of the same RF input signal. An extra DC offset voltage “1V” is applied to the modulator to set the amplifier chain gain and phase for the case that  $V_{in}$  and the sampled  $V_{out}$  are equal in phase and amplitude; this is the case for a matched termination while any mismatch or coupling from a neighbor coil will disturb the equilibrium and force the feedback loop to react. The RF signal resulting at the modulator output excites the PA final stage and closes the loop. Note that the concept of the conventional Cartesian feedback amplifier, e.g. [2], assumes constant local oscillator (LO) levels for both up- and down-conversion stages which guarantees a constant loop gain for varying RF input levels; on the contrary, in our concept the amplifier loop gain decreases with input power which improves stability at low RF input levels. Other than in conventional power amplifiers which employ a nonreciprocal circulator to separate and isolate the inner power transistor circuit from a mismatched load, in our case the amplifier transistor output voltage changes with load variations. In a first step, a Thevenin-type equivalent circuit for linear operation of the amplifier, Fig.3, was assumed in order to allow a network description of the load dependence of  $V_{out}$ ; the frequency dependence of the transfer function of the PA is represented by a band-pass filter (BPF). The behavior of our PA circuit model depends on the applied load impedance: For a matched termination of 50  $\Omega$ , no difference voltage is generated at the comparator and thus no feedback signal generated. With, e.g., a higher load impedance, the output voltage in the open loop case increases and a difference signal is obtained at the comparator; in closed-loop operation, the I- and Q- voltages from the down-converter control the up-converter in such a way that the change in output voltage is (partially) compensated, given that the phase conditions in the loop for negative feedback are satisfied.

**Analysis method for stability:** In RF feedback systems, often system stability margins are investigated using Bode plots, e.g. [3], of phase and gain of the open loop. However, a distinct decision can only be made by observing the closed-loop frequency response, i.e. the Nyquist plot. While both methods rely on representations of frequency dependence, a method more suitable for numerical evaluations is the analysis of the closed-loop transfer function of the feedback system by the root locus method. In order to decide that a system is stable or not, the root locus method develops the transfer function into a rational polynomial function of complex frequency  $s$ , with the zeros  $z_i$  in the numerator and the poles  $p_i$  in the denominator [3]. In a stable system all components of the response from a finite set of initial conditions decay to zero as time increases or  $y(t) = \lim_{t \rightarrow \infty} \sum_{i=1}^n C_i e^{p_i t} = 0$ . If any pole has a positive real part there is a component in the output that increases without bound, causing the system to be unstable. Based on the system architecture shown in Fig.2, the transfer function for each block was derived assuming fixed impedance terminations for each block. The closed loop transfer function was written with the help of equ.1 in Fig.3, determining the load dependent transfer function of the PA. As a result, the simplified closed-loop transfer function [4] of our PA model is

$$\frac{V_{out}}{V_{in}} = \frac{e^{j(\Phi_m - \Phi_d + \theta)} PA(s) \times K \times LPF(s)}{1 + e^{j(\Phi_m - \Phi_d + \theta)} PA(s) \times LPF(s) \times K \times M} + \frac{PA(s)}{1 + e^{j(\Phi_m - \Phi_d + \theta)} PA(s) \times LPF(s) \times K \times M} \quad (2),$$

where  $\Phi_m$  and  $\Phi_d$  are vector modulator and vector demodulator phase shift,  $\theta$  is phase shift of feedback pass and  $K$  is forward pass gain and  $H = K \times M$  is open loop gain and  $LPF(s)$  is low pass filter that is used to define the bandwidth of the control loop. For illustration, Fig.4 shows the closed-loop frequency response of the PA model with a set of phase settings as indicated. The closed-loop transfer function in equ.2 was programmed in MATLAB and a routine was called up in MATLAB to calculate and plot the poles and zeros of the transfer function in the complex plane.

**Results and Discussions:** Fig. 5a shows the poles and zeros of the closed-loop transfer function for 50  $\Omega$  load with the same phase shifts as in Fig. 4. Real parts of all poles are negative which predicts a stable system. Fig. 5b shows the corresponding time domain ADS simulation which exhibits a smooth exponential transition. Fig. 6 illustrates the poles and zeros of the same system terminated in 500  $\Omega$ . The positive real parts of the dominant poles predict an unstable system which is confirmed by an exponential increase seen in the time domain plot. A root locus plot can be generated by plotting the roots as a function of the PA load impedance. Observing the distance of roots to the right half-plane this allows a precise judgment of areas of stability. In a pTx array the coupled coils can generate wide variations of the load impedance seen by each PA of the array, with even negative real part, depending on the array excitation vector and the coil coupling matrix. The method allows to easily identify the stable array modes by scanning through the resulting load impedances at each PA in the pTx array.

**Reference:** [1] Solbach, K et al. ISMRM 2014, abstract 1287. [2] Dawson, J.L et al. American Control Conference 2004, Pages 361-366 vol.1. [3] Ogata, K, “Modern Control Engineering”. Prentice-Hall. [4] Gonzalez, M.A et al. EuRAD 2006, Pages 327-330.

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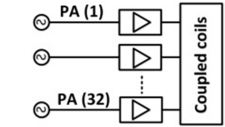


Fig.1: Array of feedback amplifier.

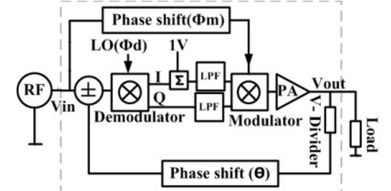
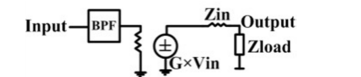


Fig.2: Simplified block diagram of Cartesian feedback PA.



$$PA(s) = G \times BPF(s) \frac{Z_{load}}{Z_{in} + Z_{load}} \quad (1)$$

Fig.3: Equivalent circuit for PA.

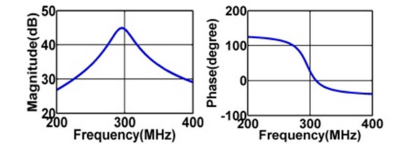


Fig.4: Closed-loop frequency response.  $\Phi_m = 0^\circ$ ,  $\Phi_d = -80^\circ$ ,  $\theta = 100^\circ$ .

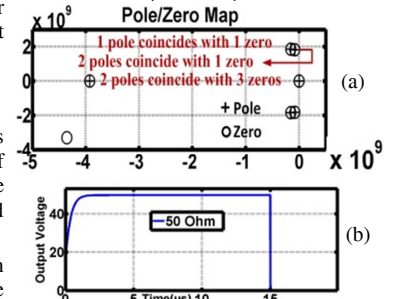


Fig.5: (a) Pole-zero plot and (b) time domain response for stable system.

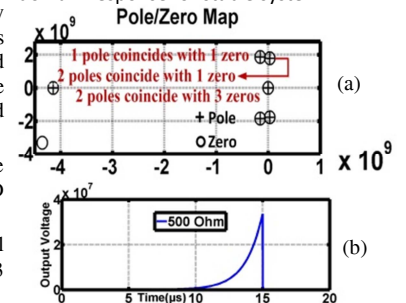


Fig.6: (a) Pole-zero plot and (b) time domain response for instable system.