Direct Derivation of Multi-Channel Receive Coil Sensitivity

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Introduction: Most of the literature on signal reception in MR treats the reception mechanism utilizing the reciprocity theorem, according to which the receive sensitivity of the channels can be assessed through the transmit sensitivity of the same very array [1]. It is stated that receive sensitivity is proportional to B_1^- fields generated by the 1A current in every loop. In modern MRI receive phased array there are additional circuits attaches to the antennas (preamp decoupling etc), so that intuitively is difficult to follow the receive path through the reciprocity theory standpoint. We propose a direct method of deriving the receive sensitivity which we find more intuitive for MRI coil signal characterization as will turn out, directly applicable when describing additional circuitry attached to the multichannel phased array.

Theory: The equations on the direct signal injection into the multichannel phased array will be derived by describing a very practical approach in coil tuning procedure – exciting the phased array with a pickup loop. Let us define a system comprising of a pickup loop and multichannel coil array. Let the pickup loop have the impedance Z_s and is excited with the voltage V. Due to mutual coupling between the pickup loop probe and the elements of the array, currents $I_1, I_2 \dots I_n$ will be induced into n element of the coil. *The problem of finding the receive sensitivity of the coil array is the problem of mapping the currents in the array as a function of the position of the probe* $I_1(\mathbf{r}_{probe}), I_2(\mathbf{r}_{probe})\dots I_n(\mathbf{r}_{probe})$. To find out the induced current into the coil array we need to know the impedance matrix of the entire ensemble $\mathbf{Z}$ and solve the problem of currents generated by the excitation in the probe

$$\begin{pmatrix} I_s \\ \mathbf{I}\end{pmatrix} = \begin{pmatrix} A & B \\ C & D\end{pmatrix}^{-1}\begin{pmatrix} V \\ \mathbf{0}\end{pmatrix}. \text{ where } \begin{matrix} A=Z_s, \quad B=\left(Z_{s1} \ \dots\ Z_{sn}\right) \\ C=\begin{pmatrix} Z_{s1}\\ \dots \\ Z_{sn}\end{pmatrix}, \quad D=\begin{pmatrix} Z_{11} & \dots & Z_{1n}\\ \dots&\dots&\dots\\ Z_{1n}&\dots&Z_{nn}\end{pmatrix}\end{matrix}, \text{ then solution } \begin{pmatrix} I_s\\ \mathbf{I}\end{pmatrix} = \begin{pmatrix} \left(A-BD^{-1}C\right)^{-1} \\ -D^{-1}C\left(A-BD^{-1}C\right)^{-1}\end{pmatrix} V \qquad (1)$$

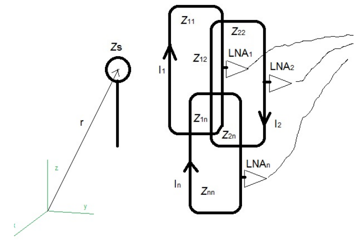

Figure 1. Pickup loop probe and RX phased array with additional electronics attached.

Let us consider here only currents induced into the coil array $\mathbf{I}$ (leaving the discussion of the current in the probe I_s out of scope, however useful in characterizing radiation damping)

$$\mathbf{I} = -D^{-1}C\left(A-BD^{-1}C\right)^{-1}V = -\begin{pmatrix} Z_{11} & \dots & Z_{1n}\\ \dots&\dots&\dots\\ Z_{1n}&\dots&Z_{nn}\end{pmatrix}^{-1}\begin{pmatrix} Z_{s1}/Z_s\\ \dots \\ Z_{sn}/Z_s\end{pmatrix}\left(1-Z_s\begin{pmatrix}\frac{Z_{s1}}{Z_s} & \dots & \frac{Z_{sn}}{Z_s}\end{pmatrix}\begin{pmatrix} Z_{11} & \dots & Z_{1n}\\ \dots&\dots&\dots\\ Z_{1n}&\dots&Z_{nn}\end{pmatrix}^{-1}\begin{pmatrix} Z_{s1}/Z_s\\ \dots \\ Z_{sn}/Z_s\end{pmatrix}\right)^{-1} V \qquad (2)$$

Equation (2) contains multiple interactions between all involved elements which can be observed in the terms of the infinite series; however it can be approximated for a probe with the impedance much smaller than impedances of the array by taking only the first term of the Taylor decomposition $(1-x)^{-1} = 1 + O[x]$. Further simplification of the equation (2) comes from the deduction of the mutual inductance between the small circular probe and an array's loop M_{si}. Utilizing definition of the curl operator through contour integral one can find that $M_{si} = \mathbf{B}_{i,1A}(\mathbf{r}_{si})\cdot\mathbf{n}_s S_s$, where $\mathbf{B}_{i,1A}(\mathbf{r}_{si})$ represents the magnetic flux density generated by 1 A in the receive array loop at the location of the probe $\mathbf{r}_{si}$ characterized by a surface area $S_s = 2\pi\rho$ and its normal vector $\mathbf{n}_s$. Considering that the impedance of the circular pickup loop is purely inductive we can use the approximation $Z_s = j\omega L_s = j\omega\mu_0\rho\cdot\text{const}$ to evaluate the ratio $Z_{s1}/Z_s = M_{s1}/L_s \sim \rho\mathbf{H}_i(\mathbf{r}_{si})\cdot\mathbf{n}_s \equiv \rho\mathbf{H}_i\cdot\mathbf{n}$, where $\mathbf{H}_i$ is the intensity of the magnetic field generated by 1A current at the location of the small circular probe of radius ρ and normal $\mathbf{n}$. With these approximations equation (2) becomes $\mathbf{I} = -\rho V D^{-1}\left(\mathbf{H}_1\cdot\mathbf{n} \quad \dots \quad \mathbf{H}_n\cdot\mathbf{n}\right)^T$. We need to mention that impedances of the array $Z_{11},\dots Z_{nn}$ include the impedances added by preamp decoupling circuitry, which if tuned and matched [4] at the resonance frequency will provide $Z_{ii} = R_{ii}(1+Z_0/R_{LNA})$, where R_{ii} is the resistance of the channel (mostly coming from the load), Z_0 characteristic impedance and R_{LNA} is the low input impedance of the preamplifier (~1-3Ohm). The current at the input of the LNA is $I_{LNA,i} = -j(\sqrt{Z_0 R_{ii}}/R_{LNA})I_i$, therefore we need to augment the equation (2) for the relationship between LNA currents and currents in the antenna loops multiplying the equation (2) by diagonal matrix $-j\sqrt{Z_0}/R_{LNA}\left[\sqrt{R_{ii}}\right]$. If the pickup loop is a quadrature type with two concentric and normal circular loop excited by voltages V and $\pm jV$ having normal vectors oriented along coordinate axes perpendicular to the main magnetic field $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ then equation (2) can be generalized as

$$\begin{pmatrix} I_{LNA,1}\\ \dots \\ I_{LNA,n}\end{pmatrix} = \rho V \begin{pmatrix} j\sqrt{Z_0 R_{11}}/R_{LNA} & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & j\sqrt{Z_0 R_{11}}/R_{LNA}\end{pmatrix}\begin{pmatrix} R_{11}\left(1+Z_0/R_{LNA}\right) & \dots & Z_{1n} \\ \dots & \dots & \dots \\ Z_{1n} & \dots & R_{nn}\left(1+Z_0/R_{LNA}\right)\end{pmatrix}^{-1}\begin{pmatrix}\mathbf{H}_1\cdot\left(\hat{\mathbf{x}}\pm i\hat{\mathbf{y}}\right)\\ \dots \\ \mathbf{H}_n\cdot\left(\hat{\mathbf{x}}\pm i\hat{\mathbf{y}}\right)\end{pmatrix} \qquad (3)$$

Discussion: It was shown how a sensitivity of the receive coil phased array can be deduced based on direct approach (vs. Reciprocity). We can trace a link between the unperturbed current in the pickup loop $I_s = V/Z_s$ and the magnetic moment of the loop $\boldsymbol{\mu}_\mathbf{n} = I_s S\mathbf{n}$ .In light of these suppositions one can see that $\rho V \sim \omega\mu_n$. The knowledge of the sensitivity derived in (3) is consistent with the formulae deriving noise covariance matrix [3]. Equation (3) shows that the receive sensitivity vector $\left[H_1^\pm \quad \dots \quad H_n^\pm\right]^T$ obtained from Reciprocity approach needs to be multiplied from left with matrices describing the coil array in cascade. In case there are variations in the LNA characteristics or differences in cable length after the LNA etc, then cascade matrices should reflect those differences.

Conclusion: Receive coil sensitivities derived in equation (3) show how RF coil electronics could change the image uniformity and SNR. It could be easily derived that Optimal SNR technique [2] is cancelling the effect of the cascade matrices leaving in the SNR product only Sensitivities from 1A current excitation and noise characterized by the inverse of the real part of the impedance matrix.

References: [1] Hoult D.I., Richards R.E., *The Signal-to-noise ratio of the nuclear magnetic resonance experiment*, J.Magn.Reson.34,425 (1976); [2] P. Roemer et al., *The NMR Phased Array,* MRM 16, 192-225 (1990);[3] R. Brown, Y.Wang, *Transmission Line Effects on the Coil Correlation Matrix in MRI, IEEE EMBS*, Vancouver, Canada 2008;[4] A. Reykowski et al., *Design of Matching Networks for Low Noise Preamplifiers*, MRM 33:848-852, 1995;