Constrained optimization of gradient waveforms for isotropic diffusion encoding

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TARGET AUDIENCE MR physicists, computer scientists, or related, interested in advanced diffusion MRI or numerical optimization of gradient waveforms.

INTRODUCTION Diffusion tensor imaging[1] (DTI) summarizes the diffusion in each voxel as a tensor with six degrees of freedom. Consequently, it requires the acquisition of at least six diffusion-weighted images. The trace of the diffusion tensor, which relates to the mean diffusivity (MD), is a useful biomarker e.g. when studying tumor cellularity[2] or diagnosing stroke[3]. It can be determined by single-shot isotropic diffusion weighting[4], i.e. without doing full DTI. Although a good idea, it has – until recently – rarely been used in practice because the limited gradient amplitudes achievable in clinical scanners have made it challenging to obtain sufficient diffusion weighting when using isotropic encoding. The recent revival stems in part from an interest in using isotropic diffusion weighting for studying microscopic diffusion anisotropy[5] and in part from developments in the numerical optimization of the gradient waveforms[6, 7]. In this work, we propose a new optimization framework for these gradient waveforms that makes far less modeling assumptions than previous work while it is at the same time easily adaptable to hardware constraint on maximum gradient amplitude, slew rate, heating and positioning of RF pulses.

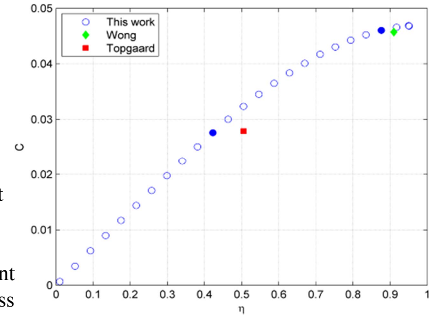

Figure 1. Sequence efficiency factor C and relative heat dissipation η for sequences optimized in this work and in previous work. The larger η is the more heat is generated by the sequence.

METHODS Instead of working directly with the gradient waveforms $g_i(t)$ $(i = x, y, z)$, we formulate the optimization problem using $q_i(t) = \gamma \int_0^t g_i(t')dt'$. The reason is that an isotropic diffusion encoding is achieved if the pulse sequence encoding $\mathbf{q}(t) = (q_x(t), q_y(t), q_z(t))^T$ satisfies[6] $\int_0^T \boldsymbol{q}(t)\boldsymbol{q}(t)^T dt = \frac{b}{3}I$, where T is the echo time and b is a scalar reflecting the strength of the diffusion encoding. Since the echo signal is given by $E(\boldsymbol{q}) = e^{-b(\boldsymbol{q})\overline{D}}$, where $\overline{D}$ is the mean diffusivity, it is natural to attempt to find a pulse sequence $\boldsymbol{q}(t)$ that maximizes $b(q)$ for a given echo time. The optimization is complicated by a number of pulse sequence- and hardware-dependent constraints. The pulse sequence-dependent constraints are first the abovementioned isotropic diffusion encoding and second the echo requirement $\boldsymbol{q}(0) = \boldsymbol{q}(T) = 0$. The hardware constraints on the maximum gradient amplitude, G_{max}, and slew rate, R_{max}, translate into componentwise constraints on the first and second derivatives of $\boldsymbol{q}(t)$. Another, possibly significant, hardware issue is heat dissipation in the gradient coils which (assuming resistive heating) is proportional to the time integral of $g_i(t)^2$. This can be captured by the constraint $\int_0^T g_i(t)^2 dt \leq \eta G_{max}^2 T$, where $\eta \in [0,1]$ is a dimensionless scalar. Varying the parameter η allows us to balance heat dissipation against diffusion encoding. Taken together, we arrive at the optimization problem:

Maximize $\quad b$

Subject to

$$\int_0^T \boldsymbol{q}(t)\boldsymbol{q}(t)^T dt = \frac{b}{3}I$$

$$\boldsymbol{q}(0) = \boldsymbol{q}(T) = 0$$

$$\frac{dq_i}{dt} \leq G_{max}, \qquad i = x, y, z$$

$$\frac{d^2 q_i}{dt^2} \leq R_{max}, \qquad i = x, y, z$$

$$\int_0^T g_i(t)^2 dt \leq \eta G_{max}^2 T, \quad i = x, y, z$$

To solve this problem we discretized $\boldsymbol{q}(t)$ and replaced the derivatives and integrals with finite difference approximations. To achieve better convergence, we also relaxed the equality in the isotropy constraint, allowing a small violation ε in Frobenius norm. These steps turn the problem into a form in which it can be efficiently solved using sequential quadratic programming. One straightforward generalization captured by this framework is to use a measurement tensor[8] other than the identity. Another generalization is to impose that the gradients must be zero during a time-interval (during which an RF-pulse is applied), which, when discretized, translates into a set of linear equality constraints.

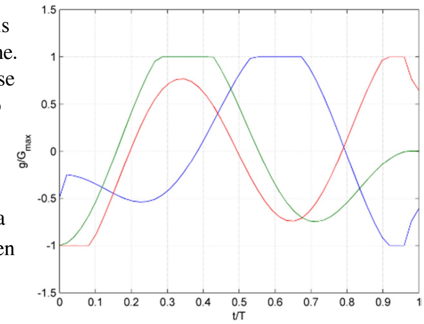

Figure 2a. Gradients optimized with $\eta = 0.42$.

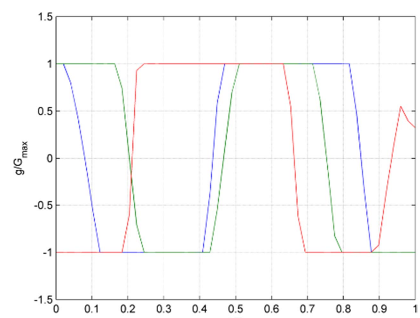

Figure 3b. Gradients optimized with $\eta = 0.88$.

RESULTS The performance of the different gradient waveforms can be compared with respect to their diffusion weighting and the amount of dissipated heat. In general, the b-value of any gradient waveform can be expressed as $b = C\gamma^2 G_{max}^2 T^3$, where C is an efficiency factor that depends on the gradient waveform. We ran optimizations using $G_{max} = 80$ mT/m, $R_{max} = 100$ mT/m, $T = 60$ ms and $\epsilon = 10^{-4}$. Figure 1 shows the efficiency C of our optimized waveforms as a function of the heat dissipation η and compares it with previous work. Figure 2a and 2b shows the optimized waveforms for the choices of η corresponding to the filled circles in figure 1.

DISCUSSION AND CONCLUSION We have proposed a new framework for optimization of gradient waveforms corresponding to a desired measurement tensor. The formulation as a constrained optimization problem allows explicit control of hardware requirements, including maximum gradient amplitude, slew rate, heating and positioning of RF pulses. The power of this approach is demonstrated by a comparison with previous work on optimization of isotropic diffusion sequences, showing possible gains in diffusion weighting or in heat dissipation, which in turn means increased signal or reduced scan-times.

REFERENCES [1] P.J. Basser et al., Biophys. J. 66 (1994). [2] L. Chen et al., PLoS ONE 8(11) (2013). [3] G. Schlaug et al., Neurology, 49(1) (1997). [4] S. Mori and P.C.M. van Zijl, MRM 33 (1995). [5] S. Eriksson et al., JMR 226 (2013). [6] E.C. Wong et al., MRM 34 (1995). [7] D. Topgaard, Micropor. Mesopor. Mat. 178 (2013). [8] Westin et al., MICCAI (2014)