Fitting the Diffusional Kurtosis Tensor to Rotated Diffusion MR Images

Pedro A. Gómez[1,2], Tim Sprenger[1,2], Marion I. Menzel[2], and Jonathan I. Sperl[2]
[1]*Technical University Munich, Munich, Germany,* [2]*GE Global Research, Munich, Germany*

Introduction: Diffusion Kurtosis Imaging (DKI) allows for the characterization of the non-Gaussian diffusion of water within a biological tissue [1]. Kurtosis is quantified by deriving scalar metrics from the fourth order kurtosis tensor, obtained in turn from fitting the measured diffusion signal to a model with 22 free parameters. Fitting this model is prone to low accuracy and high bias of the derived scalar metrics due to the low SNR of diffusion weighted images and the Rician nature of the noise distribution [2]. We hypothesize that a simpler version of the model, i.e. one that only requires the fitting of 10 free parameters, should have a lower bias and higher accuracy than the standard model. We develop this model based on the assumption that the cross-terms of the diffusion and kurtosis tensors are eliminated if the diffusion encoding space (q-space) of every voxel is rotated into the main directions of diffusion before fitting.

Theory: The second order diffusion tensor $\boldsymbol{D} \in \mathbb{R}^{3x3}$ and the fourth order kurtosis tensor $\boldsymbol{W} \in \mathbb{R}^{3x3x3x3}$ are related to the measured diffusion signal S by:

$$S(b) = S_0 \cdot \exp\left(-b \sum_{i,j} g_i g_j \boldsymbol{D}_{ij} + \frac{b^2}{6}\left(\sum_i \frac{\boldsymbol{D}_{ii}}{3}\right)^2 \sum_{i,j,k,l} g_i g_j g_k g_l \boldsymbol{W}_{ijkl}\right) \quad (1)$$

where S_0 is the non-weighted signal, b corresponds to the b-value of the diffusion experiment, and g_n represents the n-th component of the directional unit vector $\boldsymbol{g} \in \mathbb{R}^3$. We develop the reduced version of the model in three steps: first, we estimate the main directions of diffusion; second, we rotate the diffusion encoding space (q-space) of every voxel into its principal coordinates using the Eigenvectors of the diffusion tensor; and third, we fit the data in the rotated coordinate system to:

$$S(b') = S_0 \cdot \exp\left(-b' \sum_i \lambda_i + \frac{b'^2}{6}\left(\sum_i \frac{\lambda_i}{3}\right)^2 \sum_{i,j} g_{ii} g_{jj} \widehat{\boldsymbol{W}}_{ij}\right). \quad (2)$$

In (2) the cross-terms are neglected, yielding a direct calculation of the Eigenvalues λ_i of the diffusion tensor and reducing the amount of free parameters to 10: S_0, the three Eigenvalues of the diffusion tensor, and six coefficients from the kurtosis tensor, which is now second order instead of fourth.

Methods: Two experiments were performed to assess the performance of the reduced model. In the first experiment we fitted both models to a volunteer dataset. Acquisition was performed in a 3T GE MR750 clinical MR scanner (GE Healthcare, Milwaukee, WI, USA) using a 32-channel head coil (single shot EPI, single spin echo, TE =80.7 ms, TR = 1.8 s, 96x96, FOV=24 cm) and a 3-shell DKI acquisition with 25, 35 and 70 non collinear directions and corresponding b-values of 750, 1070, and 3,000 s/mm^2 [3]. Post-processing on the data included motion correction, skull extraction [4], and the estimation of rotationally invariant kurtosis metrics [5]. The second experiment consisted of a Monte Carlo simulation for three representative voxel types: one gray matter (GM) voxel and two white matter (WM) voxels, one with high and one with low fractional anisotropy (FA). The FA for each of these voxels was defined as: 0.17 for GM, 0.50 for WM with low FA, and 0.79 for WM with high FA. These three standard voxels were artificially corrupted with Rician noise to different extent and fit to both versions of the model in a linear and non-linear (NL) manner. The simulation consisted of a total of 1,000 instances for six SNR levels ranging from 10 to 20. The bias was calculated by subtracting the mean of the different instances to the ground truth voxels, while the accuracy was quantified in terms of the standard deviation (SD) of the noise instances.

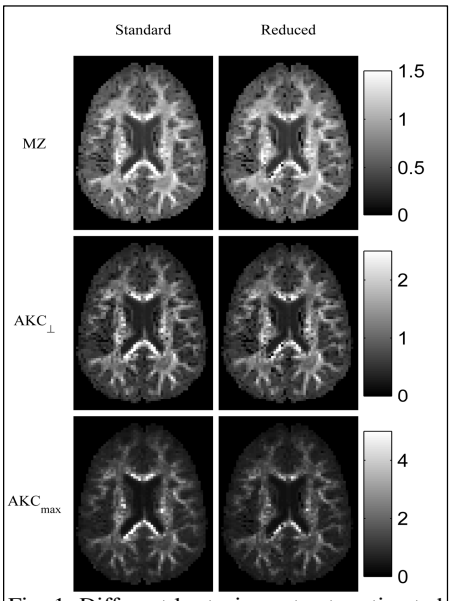

Fig. 1: Different kurtosis contrasts estimated from the standard (left column) and reduced (right column) models.

Results: Fig. 1 depicts mean kurtosis (MZ), orthogonal apparent kurtosis coefficient ($AKC_{\perp}$), and maximum apparent kurtosis coefficient (AKC_{max}) estimated from linear fitting of both the standard and reduced model. In areas of known high anisotropy, such as the areas near the corpus callosum, the estimated kurtosis maps were smaller than the maps of the standard model. Fig. 2 shows the bias and standard deviation of MZ for the different methods in the three standard voxels. In areas of WM with high FA the bias of the non-linear implementation of the reduced model was significantly less than the other implementations of the model.

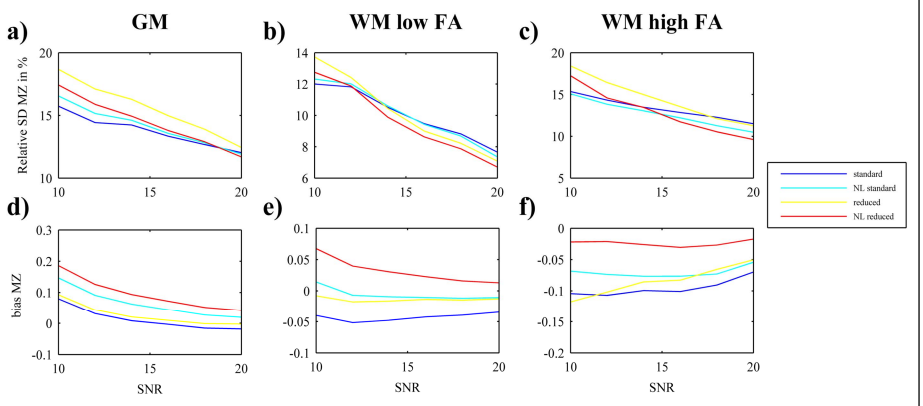

Fig. 2: Estimated standard deviation (SD) (a-c) and bias (d-f) from the Monte Carlo Simulation for characteristic a/d) gray matter (GM), b/e) white matter (WM) with low fractional anisotropy (FA), and c/f) WM with high FA voxels.

Discussion: We propose to reduce the standard kurtosis model by fitting the measured diffusion data in a rotated coordinate system. This implementation reduces the amount of free parameters from 22 to 10 by making use of an initial estimation of the main directions of diffusion. Even at a reduction of over 50% of parameters, the method yields comparable parametric maps, stability, and bias. Furthermore, results indicate that the reduced model yields parametric maps with a reduced bias in areas of high anisotropy.

References: [1] Jensen *et al.*, MRM, 2005. [2] Veraart *et al.*, MRM, 2011. [3] Poot *et al.*, IEEE Transactions on Medical Imaging, 2010. [4] Jenksinson *et al.* Neuroimage, 2012. [5] Hui *et al*. Neuroimage, 2008.