A STATISTICALLY STATIONARY ANOMALOUS DIFFUSION MODEL FOR DIFFUSION WEIGHTED IMAGING

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Target audience: Researchers with interests in non-Gaussian diffusion and the underlying microstructure of neural tissues.

Introduction: Non-monoexponential diffusion weighted signal decay in neural tissue has been linked with anomalous diffusion phenomenon [1], and experimental evidences indicate that anomalous diffusion processes in biological tissues are statistically stationary [2]. However, no theoretical model has been established for describing this phenomenon. In this study, we propose a novel anomalous diffusion model with stationary increments, which formulate the DWI signal as a stretched exponential decay that may enlighten the nature of anomalous diffusion in neural tissues.

Theory: Our theory is based on the Langevin equation $\ddot{X}(t) = -\eta\dot{X}(t) + N(t)$ and the Einstein-Smoluchowski noise paradigm $N(t) = \sum S_k \delta(t - t_k)$, where S_k is the amplitude of random shock form environment at time t_k and time series $\{t_k\}$ is a Poisson process, which makes the diffusion process statistically stationary. Using variation properties of functions, the expectation of Fourier transform of Langevin motion $X(t)$ can be computed as $\langle \exp(ikX(t)) \rangle = \exp(-D_{\alpha,\beta}|k|^{\alpha} t^{\beta})$ [3], where, $D_{\alpha,\beta}$ is fractional diffusion coefficient and the exponents α and β govern the Gaussian and Markovian properties of diffusion process, respectively. If $\alpha = 2$, the diffusion process is Gaussian, and if $\beta = 1$, it's Markovian. Here, $X(t)$ is called the fractional Lévy motion (FLM), which is typically non-Gaussian and non-Markovian. It is a statistically stationary diffusion process. According to q-space theory, when $\delta << \Delta$, the DWI signal decay was formulated as

$$S / S_0 = \exp(-D_{\alpha,\beta}|q|^{\alpha} \Delta^{\beta}) \quad (1)$$

Hence, the DWI signal decay of FLM is formulated as a stretched exponential decay.

Method: Trajectories of FLM with selected α and β values were numerically simulated based on a proposed method [4]. DWI signal decay of FLM was computed by ensemble-averaged phases based on simulated trajectories and was compared with the theoretical expectation calculated as Eq. (1). The FLM was then used to analysis in vivo DWI data. Eight healthy human subjects (4 males and 4 females) were included in this study, and consent forms were obtained. DWI images were acquired on a 3T GE MR750 scanner using a single-shot DW-EPI sequence with a 3 mm isotropic voxel size. To fit the DWI signal decay of FLM, two kinds of experiments, constant Δ value and constant q value, were performed. The maximum b-value used in constant Δ value and constant q value experiments were 3500 s/mm^2 and 3000 s/mm^2, respectively. All the obtained DWI images were first corrected for eddy currents distortions and head motion using FSL data analysis package. Then, the DWI images were spatially normalized. α and β maps of each subject were calculated by respectively fitting DWI signal decay curves of constant q and constant Δ acquisitions. Lastly, the averaged α and β values of white matter (WM), grey matter (GM), cerebrospinal fluid (CSF) and subcortical regions, such as thalamus and caudate/putamen, were extracted for each subject.

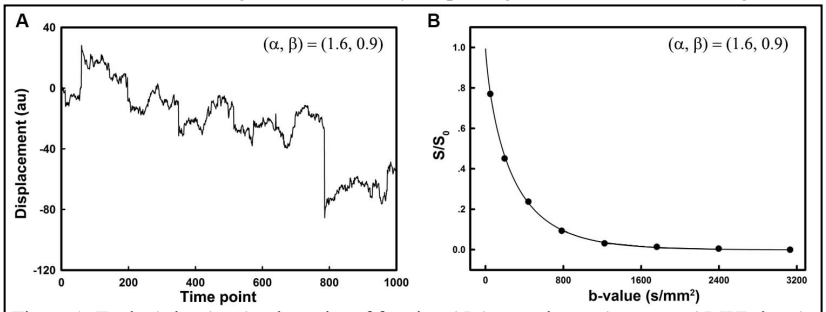

Figure 1. Typical simulated trajectories of fractional Lévy motion and computed DWI signal.

Results & Discussion: A typical one dimensional trajectory of FLM with $(\alpha, \beta) = (1.9, 0.6)$ and its related DWI signal decay are shown in Figure 1. It is seen that long jumps existed, which indicated its non-Gaussian property. The simulated signal decay matches theoretical model very well. Fitting results of a typical subject is shown in Figure 2. It is seen that both α and β maps show great contrast of the brain, and their relationship for different brain tissues are shown in Figure 3. It is seen that CSF is near to normal diffusion ($(\alpha, \beta) = (2, 1)$) and WM is more anomalous than GM because of the complexity of its underlying structure. Subcortical regions are more anomalous to GM and less anomalous than WM.

Conclusion: A statistically stationary anomalous diffusion model has been proposed in this study. It offers a theoretical foundation to stretched exponential diffusion model. Its validity was proved through numerical simulation. Great contrast in brain tissues based on FLM analysis was demonstrated in DWI data of human subjects. This model may aid further understanding of the anomalous diffusion process in neural tissues.

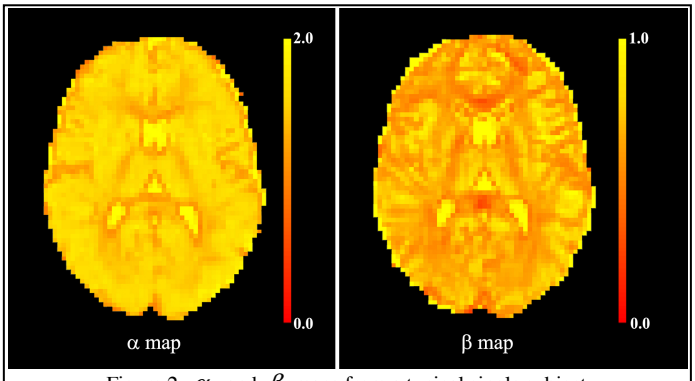

Figure 2. α and β maps from a typical single subject.

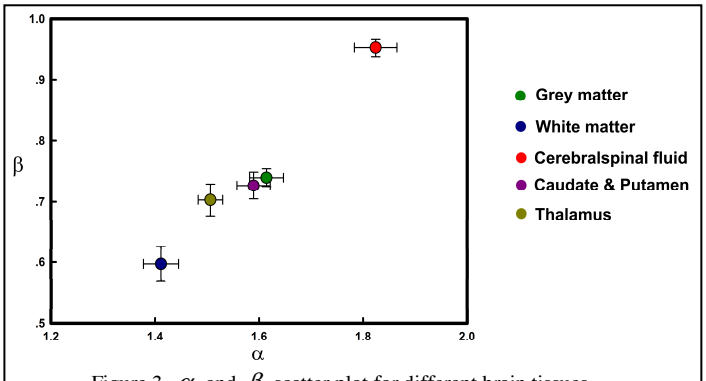

Figure 3. α and β scatter plot for different brain tissues.

References: [1] De Santis et al., MRI, 2011; [2] Magdziarz et al., PRL, 2009; [3] Eliazar et al., Phy Rep, 2013; [4] Stoev & Taqqu, Fractals, 2004.