

Line graphs and vector weights: a novel paradigm for brain network analysis

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1. Target audience: Scientists interested in brain network analysis, brain connectivity and/or graph theory algorithms.

2. Purpose: Recent years have seen major advances in research on brain networks [1]. Standard graph-theoretic brain network models consists of a set of nodes, which represent a partitioning of gray matter structures. These nodes are connected via edges that represent structural and/or functional connections between gray matter units. Such a graph model of the brain's network organization can be constructed from modalities such as structural MRI, diffusion MRI, fMRI, or EEG/MEG [1]. In this standard brain network model, a natural emphasis is placed on graph nodes which represent the brain's computational units. However, the relationship between the edges themselves is often overlooked. We introduce here a novel formalism for brain network analysis that shifts the emphasis from the nodes, which are typically gray matter structures, to edges, which represent functional or structural connectivity. This transformation makes possible the study of the organization of the connections, and the topographical relations between them. The value of this type of analysis has been demonstrated in social network analysis [2,3], but it has yet to be applied to brain network analysis. Referring to the standard brain network graph as the Primal graph, we introduce (a type of) a dual graph whose nodes correspond to the edges in the Primal graph. In the graph theory community, this construct is typically referred to as a "Line Graph". Below, we use the terms 'Line graph $L(G)$ ' and 'dual graph $D(G)$ ' interchangeably.

Methods. In standard brain network analysis, the edges in the Primal graph may have a scalar weight associated with them in order to reflect connectivity strength. It is currently not clear how to extend the Primal graph formalism to the case of vector-weighted networks. Here, we propose a two-step solution to this problem. First, we introduce a transformation from the Primal graph to a Line graph, such that weights on edges in the Primal graph become feature vectors on nodes in the Line graph. Then, we introduce a metric on vector weights associated to nodes in dual (Line graph) space. The Line graph can then be analyzed via standard graph measures, modified to take into account the metric on feature vectors associated with each node.

Consider the simple case of two feature vectors (x_1, y_1) and (x_2, y_2) . If R_x and R_y are 2 metric spaces and $x_1, x_2 \in R_x$ and $y_1, y_2 \in R_y$, then the metric for the product space $R_x \times R_y$ is given by: $d^2((x_1, y_1), (x_2, y_2)) = d^2(x_1, x_2) + d^2(y_1, y_2)$. In general, we will be working with m such feature vectors, each with n features: $f_i = (x_{i1}, x_{i2}, \dots, x_{in})$. To be able to compute distances between such feature vectors, we need to first define metrics for each individual feature space. This can be done with either an Euclidean or a Riemannian metric. In the Euclidean case, the metric is simply $d(x_{i1}, x_{j1}) = |x_{i1} - x_{j1}|$. In the Riemannian case, one possible metric is $d(x_{i1}, x_{j1}) = \log(t(x_{i1})/t(x_{j1}))$, where $t(x)$ is a mapping of the feature value from its native space to the space of non-negative real numbers. See [4] for details.

The Line graph. A network is represented as a graph $G = \{V, E\}$ where V is a set of vertices that are pairwise joined by a set of edges E . Our model is based on the concept of the Line graph, borrowed from graph theory. Given a graph G , its Line graph $L(G)$ is formed by interchanging the roles of V and E . In other words, the Line graph $L(G) = \{V_L, E_L\}$ has a one-to-one correspondence between its vertex set V_L and the edge set E of G . Furthermore, two vertices in $L(G)$ are connected if and only if the corresponding edges in G share a common endpoint (vertex). An example of this construction is shown in **Fig. 1**. The structure of the Line graph represents the organization of edges in the Primal graph. For instance, path lengths in $L(G)$ can be interpreted as distances between edge weights in G . Here, edges in G can be weighted by vectors of features, not just scalars as typically done in brain connectivity literature. See [2,3,4] for details.

4. Experiments To illustrate the method's application to neuroscientific studies, we performed a small scale population study on 14 adult male controls and 10 adult male chronic schizophrenia subjects, focusing on structural connectivity derived from diffusion MRI. In each subject, we computed whole-brain tractography, followed by a standard FreeSurfer parcellation of the cortex which was then registered to the subject's diffusion MRI space. From the whole-brain tractography we extracted all existing connections between pairs of FreeSurfer-defined cortical areas. As the number of fibers per connection varies across subjects, we selected those connections that have at least 30 fibers in every subject. There were 218 such connections. Based on them, we constructed a structural network graph, our Primal graph G , shown in **Fig. 2**. Three features were computed for each connection in this network: mean tensor FA, mean tensor Trace, and mean Dispersion. The latter is a geometrical descriptor defined in [5]. We then computed $L(G)$ as described previously. Each node in $L(G)$ represents one of the 218 connections and is associated with a feature vector of three elements, i.e. the mean values of FA, Trace and Dispersion. Using the distance measures defined above, we then computed the global efficiency measure on the Line graph $L(G)$, and performed a group comparisons. This is one possible network measure that can be used to compare graphs. Here, we restrict ourselves to one measure for brevity of exposition. Also, we note that previous network analysis studies have shown global efficiency in functional networks to be abnormal in schizophrenia.

5. Results Table 1 shows p-values for the global network efficiency comparison between our schizophrenia and controls groups, performed on both the Primal and Line graphs. Column heading $D(G)$ indicates efficiency was computed on the Line graph (or dual graph, $D(G)$), with the Euclidean distance measure (d_E) or the Riemannian distance measure (d_R). Column heading G indicates efficiency was computed on the Primal graph G . FA, Tr, Disp indicate the features used in the feature vector when working with the Line graph, or alternatively indicate the scalar weight used in the Primal graph G . These features were used either alone, as scalar weights, or together in a vector weight (last line in Table 1). Significant p-values (below 0.05) are indicated in boldface.

6. Discussion As a preliminary study, the goal of our experiments is not to make clinical claims of significance in schizophrenia, but rather to illustrate the method and to suggest ways in which it could be used in the future. Our experiments demonstrate the benefits of the Line graph framework. First, we are no longer restricted to single scalar features (weights) on the Primal graph. We can now work with feature vectors - the equivalent of vector-valued edge weights in the Primal graph. Second, even when using single scalar features, the Line graph formalism appears to allow for a better discrimination between the control and schizophrenia groups. The importance of Line graphs is known in the graph theory community, in particular in social network analysis [2,3]. We believe that it is an important tool for describing brain networks as well, and the present work is only the first step in introducing this tool to the brain network analysis community. Future work will investigate how the Line graph formalism can be used to integrate information from functional and structural connectivity into a single analysis framework.

7. References. [1] Bullmore ET, Sporns O. *Nature Rev Neurosci* 10, 186–198 (2009). [2] Evans TS, Lambiotte R. *Phys.Rev.E* 80 016105 (2009). [3] Evans TS, Lambiotte R, *Eur. Phys. J. B* 77, 265–272 (2010). [4] Savadjiev P. et al., *Proc. CDMRI* 2014. [5] Savadjiev P et al., *Medical Image Analysis* 18:1349–60 (2014).

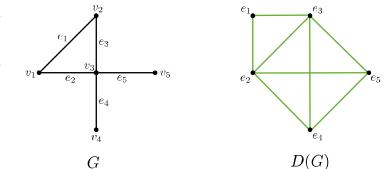


Figure 1 An example of a primal graph G and its line graph, or dual graph, $D(G)$.

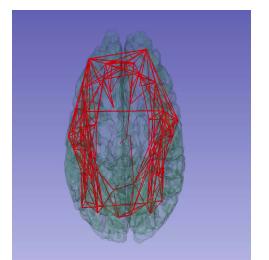


Figure 2 Our structural network consisting of 218 connections.

	$D(G), d_E$	$D(G), d_R$	G
FA	0.92	0.83	0.58
Tr	0.0075	0.0075	0.65
Disp	0.15	0.026	0.19
(FA, Tr, Disp)	0.36	0.063	

Table 1 Our group comparison experiments, summarized as p-values. See Section 5.