Multi-Kernel Estimation of Fiber Orientation Distribution Functions With L0-Norm Induced Group Sparsity

Pew-Thian Yap[1], Yong Zhang[2], and Dinggang Shen[1]
[1]Department of Radiology, University of North Carolina, Chapel Hill, North Carolina, United States, [2]Department of Psychiatry & Behavioral Sciences, Stanford University, California, United States

Target Audience: Diffusion MRI researchers who are interested in estimating local fiber orientations accurately with explicit consideration of different diffusion compartments with spatially varying parameters.

Purpose: Spherical deconvolution (SD) of the diffusion-attenuated signal with a fiber kernel, which represents the signal response of a single coherent fiber bundle, has been shown to yield high-quality estimates of fiber orientation distribution functions (FODFs) [1]. However, an inherent limitation of this approach is that the fiber kernel is assumed to be *invariant* throughout the image. This has been shown to result in spurious FODF peaks as the discrepancy of the fiber kernel with the data increases [2]. The SD approach has been recently extended [3] to include kernel functions of not only the white matter (WM), but also the gray matter (GM) and the cerebrospinal fluid (CSF). However, the same problem persists because the kernels, once their parameters have been determined, are also fixed and assumed to be invariant. We propose here a method to use collaboratively multiple fiber kernels for robust FODF estimation.

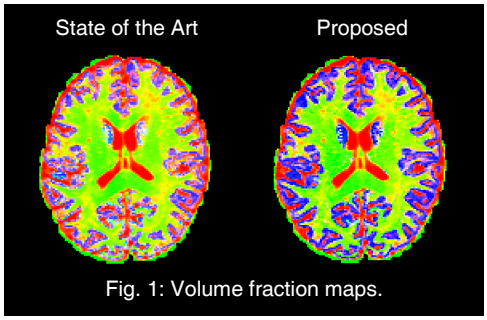

Fig. 1: Volume fraction maps.

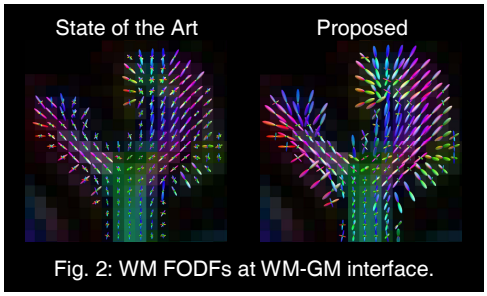

Fig. 2: WM FODFs at WM-GM interface.

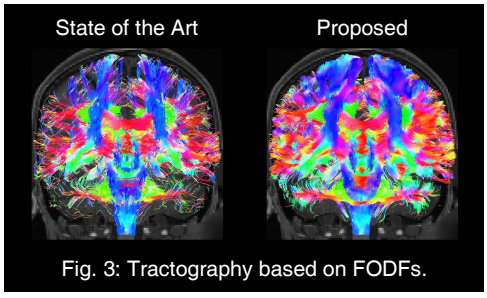

Fig. 3: Tractography based on FODFs.

Methods: *Data* – For reproducibility, diffusion weighted (DW) data from the Human Connectome Project [4] were used. The 1.25 × 1.25 × 1.25 mm^3 data were acquired with diffusion weightings of b = 1000, 2000, and 3000 s/mm^2 each applied in 90 directions. 18 baseline images with low diffusion weighting were also acquired. All images were acquired with reversed phase encoding for correction of EPI distortion. *FODF Estimation* – We estimate the FODF by using multiple *groups* of kernels (a group each for a WM direction, GM, and CSF) via solving an L0-norm group-penalized least-squares problem. Each WM group consists of unidirectional axial-symmetric diffusion tensors with a range of possible axial and radial diffusivities. The GM and CSF groups consist of isotropic tensors with diffusivities of GM kernels set lower compared with CSF, consistent with what was observed in [3]. These kernel functions are sampled in a number of directions and diffusion weightings according to the sampling pattern of the DW data and are collected as columns in a matrix A. The FODF for each voxel is then estimated by solving $\min_f \|Af - s\|_2^2 + \lambda_1 \sum_{g\in G} I(\|f_{J(g)}\|_2) + \lambda_2 \|f\|_0$, where s is the signal vector, f is a vector representing the volume fractions associated with the kernel functions, $J(g)$ is the set of indices of columns in A that correspond to the kernels in group g, $I(z)$ is an indicator function that returns 1 when z is non-zero, and λ_1 and λ_2 are the tuning parameters. We define $\mathrm{G} = \{\mathrm{WM}_1, \mathrm{WM}_2, \ldots, \mathrm{WM}_N, \mathrm{GM}, \mathrm{CSF}\}$, where WM_k indicates a group of anisotropic tensors oriented along the k-th sampling direction of the FODF. The FODF, discretized in N directions, is computed simply by summing the volume fractions of the kernels within each of the N WM groups. The optimization problem is solved using iterative hard thresholding [5].

Results: We compared the proposed method with a state-of-the-art method [6] based on reweighted L1 minimization (approximates L0 minimization). Note that we solve an L1-norm *penalized* problem instead of a *constrained* problem as in [6] to avoid having to explicitly select the sparsity to limit the number of active compartments. The compartment volume fraction maps (Fig. 1) indicate that the proposed method yields cleaner compartment estimates especially for cortical gray matter. In contrast, the competing method gives results that are contaminated, especially by the CSF compartment. This contamination results in inaccurate WM FODF estimates in the WM-GM interface (Fig. 2), causing false-positive peaks and hence greater directional uncertainty. Accumulation of FODF errors results in errors and irregularities in tractography (Fig. 3).

Discussion: We have shown that instead of restricting ourselves to one kernel per compartment, it is possible to employ a group of kernels per compartment to cater to possible data variation across voxels. The use of L0-norm penalization is motivated by the following observations: (1) L1-norm penalization conflicts with the unit sum requirement of the volume fractions, as noted in [6]; (2) L1-norm penalization is biased and attenuates coefficient magnitudes [7], resulting in the erosion of FODF peaks. Our results confirm our observations and demonstrate that the proposed method improves microstructural and tract estimates.

Conclusion: Our method provides the flexibility of including different diffusion models in different groupings for robust microstructural estimation. Future work includes incorporating complex diffusion models [8] for estimation of subtle properties such as axonal diameter. We will also apply our method for investigation of pathological conditions such as edema.

References: [1] Tournier et al., NeuroImage 35, 1459-1472, 2007. [2] Parker et al., NeuroImage 65, 433–448, 2013. [3] Jeurissen et al., NeuroImage, 103, 411–426, 2014. [4] Van Essen et al., NeuroImage, 80, 62–79, 2013. [5] Z. Lu, Mathematical Programming 147, 125-154, 2014. [6] Daducci et al., Medical Image Analysis, 18, 820–833, 2014. [7] Wright et al., IEEE Trans. Signal Processing, 57, 2479-2493, 2009. [8] Panagiotaki et al., NeuroImage, 59, 2241–2254, 2012.