LASADD: Linear Acceleration Method for Adapting Diffusion Dictionaries

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Purpose: Diffusion Tensor (DT) Imaging enables the study of brain connectivity, which has presented problems in estimating the intra-voxel orientations, especially in the case of crossing fibers. We propose a Linear Acceleration of Sparse and Adaptive Diffusion Dictionary (LASADD) that models more properly the intra-voxel information. Based on the Diffusion Basis Functions Model (DBF) [1], we propose to dynamically adapt the dictionary of diffusion functions by changing the size and orientation of each active DT. The corresponding optimizations are reduced to simple non-negative least squares and linear least squares problems that iteratively achieve a better fit.

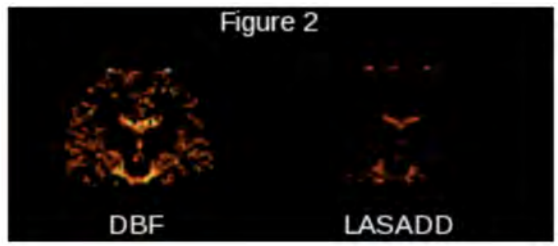
Methods: Diffusion multi-fiber can be modeled as a linear combination of Gaussian [2]: $S_i = \sum_j \alpha_j \varphi_{ij}$, with $\varphi_{ij} = S_0 \exp(-b_i g_i^T T_j g_i)$. Each tract is associated with a DT with dominant orientation through the eigenvector with the biggest eigenvalue. We use the formulation $T_j = \lambda_j^1 v_j v_j^T + \lambda_j^2 I$ to define each basis tensor, where v_j is the corresponding Principal Diffusion Direction (PDD). DBF consists on to find the linear combination of basis functions that best approximates the signal. Based on that, here arises the following adaptive algorithm:

1. Given a DBF, select the "candidates" BF of the dictionary by minimizing a non-negative least squares problem with a minimum L1-norm penalization (LASSO); i.e. a space solution.
2. Upgrade the alpha coefficients of the "active" basis functions, without the weighted penalty. Then a) seeks a small rotation in the PDD of each DT by computing a residual vector δ_j such that $v_j^* = v_j + \delta_j$. To solve this problem, we reduce the exponential function to its Taylor series of the first order, which leads to finding $\min_\delta \|R + \Omega\delta\|_2^2 \;\; s.t. \; \|\delta_j\|_2^2 \le \theta$, where $R = S - \Phi\alpha$ and $\Omega_{ij} = 2\alpha_j \varphi_{ij} \lambda_j^1 b_i v_j^T g_i g_i^T$. b) Then, we adjust the size of the PDD in the active tensors with respect to the rotation performed so that its PDD will be unitary, therefore $\lambda_j^{1^*} = \lambda_j^1 \|v_j + \delta_j\|_2^2$. c) Upgrade the tensor profiles, it seeks the modification τ_j such that $\lambda_j^{2^*} = \lambda_j^2 + \tau_j$. Similarly proposes a reduction of the exponential to its Taylor series of the first order to solve $\min_\delta \|R + \mathrm{X}\tau\|_2^2 \;\; s.t. \; \lambda^T\lambda + \tau \ge 0$, where $\mathrm{X}_{ij} = \alpha_j \varphi_{ij} b_i g_i^T g_i$. This phase is iterated until it converges to a constant dictionary.
3. Finally, the combinations of 2 and 3 basis functions are added to the "active" dictionary to return to step 1.

This algorithm takes a fixed number of iterations or until it converges. To reduce the computational cost, we use Projected Gauss-Seidel Method to select the basis functions. It is important to note that unlike the method in [3], the optimization subproblems in the step 2.1 and 2.3 are quadratic programs. In our experiments we found that the solutions to steps 2.1 and 2.3 can be approximated by simple clip of the linear least squares solution.

Figure 1

Metric	LASADD	DBF
Mean Reconstruction Error	0.026960	0.194733
SD Reconstruction Error	0.044236	0.132071
Mean Angular Error	7.802226	8.224753
SD Angular Error	6.705088	5.552135
Mean Alpha Error	0.267395	0.285921
SD Alpha Error	0.203273	0.216684
Mean Lambda 1 Error	0.181522	0.364555
SD Lambda 1 Error	0.129973	0.105946
Mean Lambda 2 Error	0.201109	0.367167
SD Lambda 2 Error	0.149402	0.308758
%Underestimation	38.283828	47.304730
%Overestimation	6.050605	6.270627



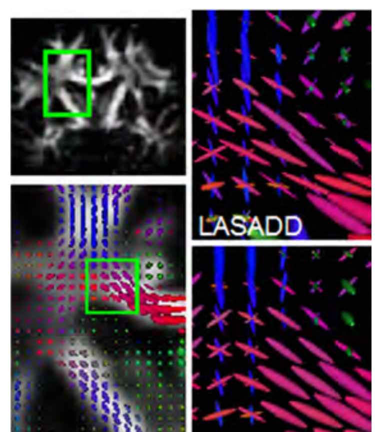


Figure 3

Experiments and Results: We compare the estimations of our proposal and DBF. For this reason, we simulate a synthetic multi-shell data by using the CUPS65 protocol [4] with b values, 2000 and 2500 with 30 gradients each one, and four images with b=0. Then, Rician noise was added to each measurement to produce a SNR=20. Fig. 1 shows the error measurements for 910 voxels. Our proposal presents lower error. For in vivo experiments, a single healthy volunteer was scanned on a Philips Achieva TX 3.0 scanner with 16 channels. We acquire 4 B0 images and 64 multi-shell DW images with b = 2000 and 2500 and a SNR=30. We see the reconstruction error or quadratic norm of the residue in Fig. 2 for our proposal and DBF. Fig. 3 depicts the estimations in one zone with well-known specific crossing. Note that our method computes better estimations than DBF.

Conclusions: We presented a set of improvements to the algorithm that adapts the dictionary of diffusion functions in rotations and size of the diffusion tensor profile. Our proposal simplifies the optimization problems using linear approximations in the rotations and size of diffusion profiles are estimated solving simple linear least-squares sub-problems. We evaluated our method performance with benchmark dataset and demonstrated it capabilities using real data.

References: [1] Ramirez-Manzanares A. IEEE-TMI, 2007. 26(8), 1091-1102. [2] Tuch D.S. MRM, 2002. 48, 577-582. [3] Aranda, R *et al* to ISMRM 2015. [4] Scherrer, PloS ONE, 2012. 7(11).