Model-based diffusion tensor denoising with tensor and FA smoothness constraints

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INTRODUCTION: Low Signal-to-noise ratio is a significant problem in diffusion tensor imaging in various applications. Recently, sparse\smooth, low rank and edge constraint models [1-5] have been successfully applied in denoising DWIs. However, these methods focused on acquiring denoised images. One has to go through an estimation chain (i.e., image→ tensor→ eigen-value→FA) to get the clinic valuable FA map. There may be severe error propagation during this nonlinear process. In this work, we propose to use the model-based method [6-7] with parametric smoothness constraints for DTI denoising. Similar works have also been developed for sparse reconstruction [8-12]. Notably, we creatively penalize the non-smoothness of the nonlinear FA map. To enable this, we manage to calculate the FA values from elements of the diffusion tensor directly without computing the eigen-values. Experiments based on a DTI brain phantom were conducted to demonstrate the feasibility of the proposed scheme in heavy noise case.

THEORY AND METHOD: The signal acquisition scheme of the diffusion weighted image at the m -th direction can be expressed as $\mathbf{d}_m = \mathbf{F}\boldsymbol{\rho}_m + \mathbf{n}_m$, where $\boldsymbol{\rho}_m \in \mathbb{C}^{N\times 1}$ is the diffusion weighted image with N the number of image pixels, $\mathbf{F}$ denotes Fourier encoding, $\mathbf{n}_m$ is measurement noise. Given the signal model $\rho_m = I_0 e^{i\boldsymbol{\varphi}_m} e^{-b\mathbf{g}_m^T \mathbf{D} \mathbf{g}_m}$ (i.e., φ_m the image phase, I_0 the reference image without any diffusion, $\mathbf{g}_m = \left(g_{xm}, g_{ym}, g_{zm}\right)^T$ the diffusion gradient vector) and the assumption that $\mathbf{n}_m$ is complex Gaussian, we propose to denoise the diffusion tensors $\mathbf{D}$ in a penalized maximum likelihood formulation incorporating the smoothness constraints (i.e., total variation) of the tensor and the FA maps simultaneously:

$$\hat{\mathbf{D}} = \arg\min_{\mathbf{D}} \sum_m \left\| \mathbf{d}_m - \mathbf{F} I_0 e^{i\boldsymbol{\varphi}_m} e^{-b\mathbf{g}_m^T \mathbf{D} \mathbf{g}_m} \right\|_2^2 + \gamma_1 \sum_{k=1}^{6} \mathrm{TV}(\mathbf{D}_k) + \gamma_2 \mathrm{TV}(\mathbf{FA}) \quad (1),$$

where $\mathbf{D} \in \Re^{N\times 6}$ consists of 3×3 symmetric tensors $D = \begin{pmatrix} D_1 & D_4 & D_5 \\ D_4 & D_2 & D_6 \\ D_5 & D_6 & D_3 \end{pmatrix}$ at each spatial location. To solve Eq. (1), we have proven that the FA actually is a non-linear function of the diffusion tensor on a pixel-by-pixel basis:

$$\mathrm{FA} = \frac{\sqrt{2}}{2} \sqrt{3 - \frac{\left(D_1 + D_2 + D_3\right)^2}{D_1^2 + D_2^2 + D_3^2 + 2D_4^2 + 2D_5^2 + 2D_6^2}} \quad (2)$$

Thus, we managed to formulate FA without calculating the eigen-values and standard non-linear conjugate gradient method can be easily adopted.

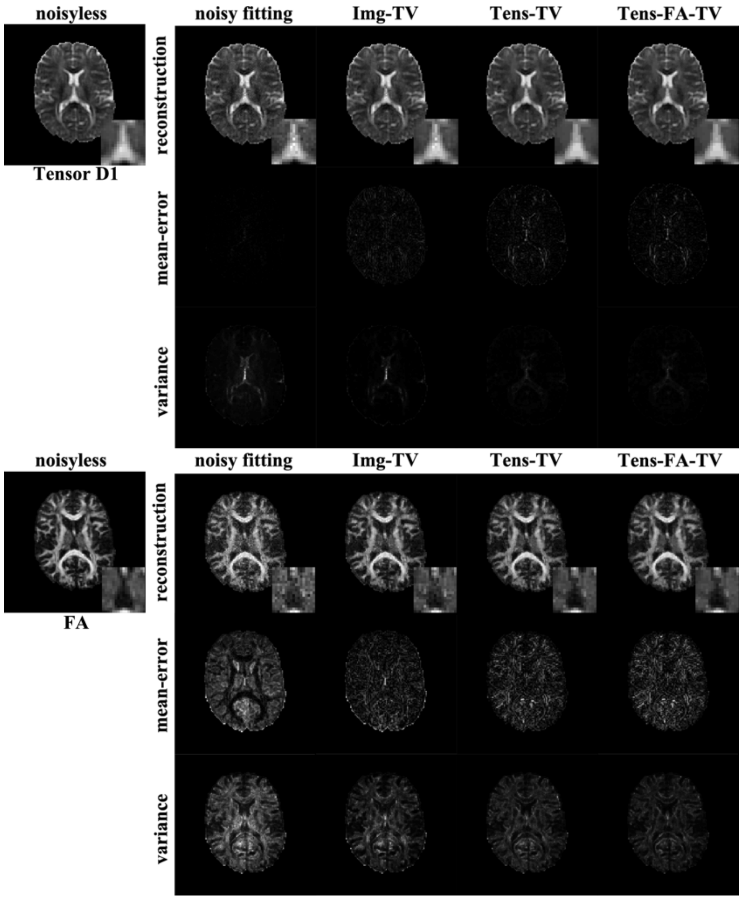

Fig. 1 Model-based denoising (Up: an example of diffusion tensor (D_1); Bottom: the FA map) from a Monte Carol study at a SNR of 5 (signal mean / noise standard deviation) with 100 noise realization. Smoothness constraints were enforced in different domains.

EXPERIMENT AND RESULT: We conducted the model-based denoising method with smoothness enforced in different domains (i.e., image, diffusion tensor, tensor and FA jointly) to validate the proposed scheme. A DTI brain phantom was used to simulate a set of diffusion weighted images (matrix size 128×128) with six diffusion directions. Monte Carol study was conducted with 100 noise realizations each mimicking a SNR of 5 (signal mean / noise standard deviation) noise level. Denoising results are shown in Fig. 1. As can be seen, for the tensor map, methods directly imposing smoothness in the tensor provides better denoising results than that promoting smoothness in the image domain. We can clearly see that the "Img-TV" method results in some singular point in the tensor map. We also observed that the superiority in denoising is accordant with the variance map (i.e., predominant in MSE) while in contrast to the mean error, which reveals the trade-off relationship between the variance and the bias of the denoiser. For the FA map, similar conclusion can be made. The "Img-TV" method results in large variance and noisy FA map. The "Tens-TV" method provides better results due to its imposing smoothness in a more "forward" domain. The "Tens-FA-TV" method, with additional smoothing in the FA domain, pays back the clearnest FA map than the other two methods and the variance map is accordingly the smallest. Note that the ML estimator (noisy fitting) is approximately unbiased in terms of tensor denoising but become biased for the FA map due to its nonlinearity.

CONCLUSION: In this work, we investigated the model-based method in DTI denoising and verified that directly imposing smoothness\sparsity in diffusion tensor and FA domain jointly will provide the best denoising performance.

ACKNOWLEDGEMENT: We would like to acknowledge NSFC 11301508, 61471350, 61102043, 81120108012 and the Shenzhen Peacock Plan KQCX20120816155710259.

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