4D Flow Imaging Incorporating a Fluid Dynamics Model

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INTRODUCTION

Cardiovascular MRI is commonly accelerated using image models that enforce general mathematical properties of cardiac images (e.g., sparsity, low-rankness) to enable sparse sampling of $(\mathbf{k}, t)$-space. Physics-based models (constraints) complement existing sparsity and low-rankness constraints and can further enhance cardiovascular imaging in terms of speed and reconstruction quality. In this work, we accelerate 4D flow imaging using an image model incorporating conservation of mass, conservation of momentum, and known physical transport properties of blood. This image model is generated by integrating computational fluid dynamics (CFD) into image reconstruction: we solve the Navier-Stokes equations—with boundary conditions reconstructed from limited $(\mathbf{k}, t)$-space data—and reconstruct 4D flow-sensitive images using the CFD solution as a constraint.

METHODS

In vivo data were collected on a Siemens TRIO 3 T scanner using ECG gating and respiratory navigation. A sagittal 3D volume was imaged using the method described in [1] with temporal resolution = 40.8 ms, cardiac phases = 21, T_R = 5.1 ms, T_E = 2.4 ms, FA = 7°, FOV = 250 mm × 330 mm × 55 mm, matrix size = 120 × 160 × 26, spatial resolution = 2.1 mm × 2.1 mm × 2.1 mm, and V_{enc} = 1.5 m/s. Reference images and velocity-encoded images in three directions (i.e., images acquired with referenced four-point encoding) were densely sampled in $(\mathbf{k}, t)$-space, resulting in a total scan time of 44 min.

To evaluate the proposed method, all data were retrospectively undersampled in k_x, k_y, and t according to a uniform random distribution, except for a 5 × 5 region at the center of (k_x, k_y)-space that was densely sampled to allow temporal subspace estimation [2]. Velocity-encoded data were undersampled by a factor of 12, and reference data were undersampled by a factor of 4, resulting in a total acceleration factor of 8 and an equivalent scan time of 5.5 min.

Reference images $\rho_{\text{ref}}(\mathbf{r}, t)$ were reconstructed using joint sparsity and low-rank/subspace constraints [3]: i.e., according to Eq. 1, where $\mathbf{d}_{\text{ref}}$ is the sparsely sampled reference data, S_{ref} is the estimated temporal subspace, Ω is the sparse sampling operator, and T is a sparsifying transform. Flow regions were identified from the angiogram $b(\mathbf{r}) = \sqrt{\sum_i |\bar{\rho}_i(\mathbf{r}) - \bar{\rho}_{\text{ref}}(\mathbf{r})|^2}$, where $\bar{\rho}_{\text{ref}}(\mathbf{r})$ is the temporal mean of the reconstructed reference images and where $\{\bar{\rho}_i(\mathbf{r})\}_{i=1}^3$ (the temporal means of the velocity-encoded images) were reconstructed using a sparsity constraint on $b(\mathbf{r})$ (i.e., according to Eq. 2).

$$\arg\min_{\rho_{\text{ref}}(\mathbf{r},t)\in S_{\text{ref}}} \sum_i \left\| \mathbf{d}_{\text{ref}} - \Omega\{\mathcal{F}_{\mathbf{r}}\{\rho_{\text{ref}}(\mathbf{r},t)\}\} \right\|_2^2 + \lambda \|T\{\rho_{\text{ref}}(\mathbf{r},t)\}\|_1 \quad (1)$$

$$\arg\min_{\{\bar{\rho}_i(\mathbf{r})\}_{i=1}^3} \sum_{i=1}^3 \left\| \bar{\mathbf{d}}_i - \bar{\Omega}\{\mathcal{F}_{\mathbf{r}}\{\bar{\rho}_i(\mathbf{r})\}\} \right\|_2^2 + \lambda \sum_m \sqrt{\sum_{i=1}^3 |\bar{\rho}_i(\mathbf{r}_m) - \bar{\rho}_{\text{ref}}(\mathbf{r}_m)|^2} \quad (2)$$

We then used CFD to generate priors for the 4D velocity maps. We constructed a 3D mesh of the vessel walls from the angiogram $b(\mathbf{r})$ using ParaView (Kitware Inc.), which served to define no-slip boundary conditions in our flow model. Velocity maps from one axial slice (analogous to one sequence of 2D velocity-encoded images) were incorporated into the model as additional boundary conditions. The kinematic blood viscosity was assumed to be ν = 3.3 mm^2/s, and each outlet was modeled to have a constant pressure gradient. Full 4D velocity maps $\{v_{\text{mod},i}(\mathbf{r}, t)\}_{i=1}^3$ were generated by solving the Navier-Stokes equations for unsteady, incompressible, laminar flow in OpenFOAM (OpenCFD Ltd.).

Finally, we reconstructed each velocity-encoded image $\rho_i(\mathbf{r}, t)$ using $\rho_{\text{mod},i}(\mathbf{r}, t) = \rho_{\text{ref}}(\mathbf{r}, t)\exp\left(j\pi v_{\text{mod},i}(\mathbf{r}, t)/V_{\text{enc}}\right)$ as prior information:

$$\arg\min_{\rho_i(\mathbf{r},t)\in S_i} \sum_i \left\| \mathbf{d}_i - \Omega\{\mathcal{F}_{\mathbf{r}}\{\rho_{\text{ref}}(\mathbf{r},t)\}\} \right\|_2^2 + \lambda \left\| T\{\rho_i(\mathbf{r},t) - \rho_{\text{mod},i}(\mathbf{r},t)\} \right\|_1 \quad (3)$$

RESULTS

Figure 1 shows boundary condition geometry defined from $b(\mathbf{r})$ and the single-slice 2D images. Figure 2 shows example slices of v_z maps from the fully-sampled scenario, from a low-rank- and sparsity-constrained reconstruction of the sparse data, and from a reconstruction using the proposed method. Figure 3 shows pathlines calculated from each set of results using ParaView. Where relevant, the results in all figures used rank = 11 and the sparsifying transform $T = \mathcal{F}_t$ (the temporal Fourier transform).

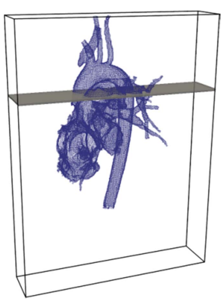

Fig. 1: Example boundary condition geometry

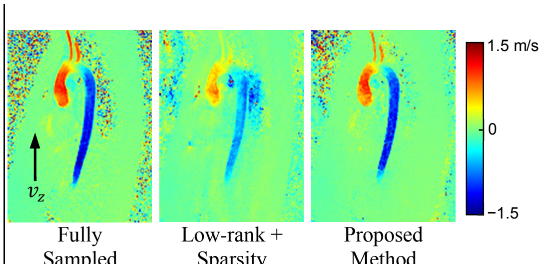

Fig. 2: Example slices from v_z maps. The proposed method reconstructed high-quality images from 1/12 of the velocity-encoded data and 1/4 of the reference data.

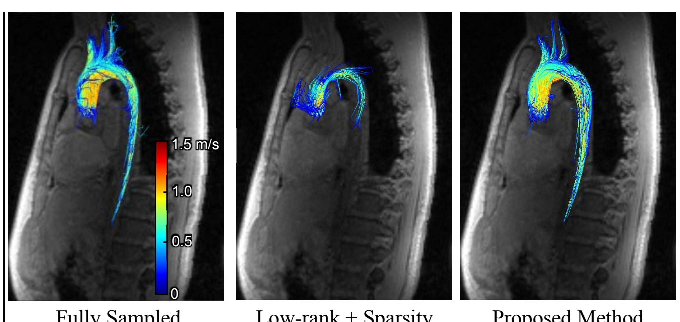

Fig. 3: Example pathlines at one time point

CONCLUSION

We present a novel approach to high-speed 4D flow imaging incorporating a physics-based flow model. Our preliminary results indicate that the proposed method far outperforms state-of-the-art methods enforcing mathematical properties (such as sparsity and low-rankness) alone. The proposed method's equivalent scan time of 5.5 min represents a significant time savings over the fully sampled scenario's 44 min, but even further acceleration may be possible by leveraging parallel imaging or with improved flow models (e.g., models incorporating turbulence and/or patient-specific viscosity measurements).

REFERENCES [1] M. Markl *et al.*, *JMRI* 2007, pp. 824-31. [2] Z.-P. Liang, *IEEE-ISBI* 2007, pp. 988-91. [3] B. Zhao, *et al.*, *IEEE-TMI* 2012, pp. 1809-20.