Image Reconstruction for Accelerated Diffusion Tensor Imaging Using Joint Low-Rank and Sparsity Constraints

Sen Ma[1], Xiaodong Ma[2], and Hua Guo[2]

[1]*Department of Electronic Engineering, Tsinghua University, Beijing, China,* [2]*Center for Biomedical Imaging Research, Department of Biomedical Engineering, School of Medicine, Tsinghua University, Beijing, China*

Introduction: Diffusion tensor imaging (DTI) has been widely used for revealing microstructures of biological tissues, and is often applied to clinical studies like fiber tracking and white matter mapping [1,2]. A significant issue of DTI is the extensively long examination time when the repetitive acquisition pattern is performed across multiple diffusion directions. In this work, we propose an effective joint reconstruction method to accelerate DTI acquisition, combining the low-rank (LR) structure and sparsity constraints of the correlated diffusion-weighted images. LR-based methods have been previously used for compressive diffusion-weighted MRI [3] and denoising of DTI [4]. Here we show that by jointly enforcing LR and sparsity constraints, we can achieve high reduction factor of DTI acquisition while maintaining rather accurate reconstruction result.

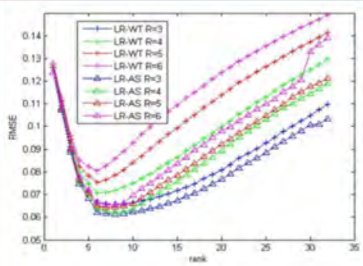

Fig 1. Quantitive comparison of RMSE for LR-WT and LR-AS from rank=1 to 32.

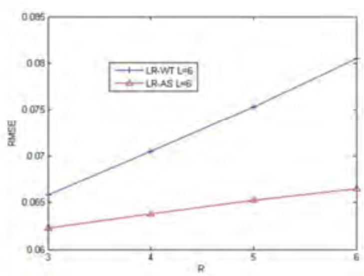

Fig 2. Quantitive comparison of RMSE for LR-WT and LR-AS at L=6 of R=3,4,5,6

Methods: The LR structure of diffusion tensor images is due to (1) strongly correlated spatial-diffusion behaviors of noiseless diffusion signals at different voxels and (2) low degrees of freedom in the image tensor model [4]. Given a Casorati matrix $\mathbf{C} = [\rho_1, \rho_2, ..., \rho_N]$ where the magnitude diffusion-weighted image of direction i is reshaped into a vector ρ_i, the low-rank structure can be represented as $\mathbf{C} = \mathbf{UV}$ based on a partial separability model where $\mathbf{V} \in \mathbb{C}^{L\times N}$ spans the "diffusion subspace" of $\mathbf{C}$ and $\mathbf{U} \in \mathbb{C}^{M\times L}$ contains the corresponding spatial coefficients [5]. Both $\mathbf{U}$ and $\mathbf{V}$ are low-rank matrices with $L \ll N < M$. Furthermore, we may enforce an even stronger low-rank constraint by predetermining $\mathbf{V}$ from the full sampled navigator at the center of k-space using singular value decomposition (SVD) [6]. This allows us to reduce our reconstruction problem to the determination of $\mathbf{U}$. In addition to the low-rank constraint, $\mathbf{C}$ has sparse representations in certain transform domains. Here we compare two kinds of sparsity: (1) the conventional wavelet transform (WT) sparsity and (2) the anisotropic sparsity (AS) [7] induced by the similarity of DWIs of multiple directions due to the isotropic diffusion process, respectively. For these two sparsity constraints, the image reconstruction problem can be formulated as:

$$\hat{\mathbf{U}} = \arg\min \|\mathbf{d} - \Omega\{\mathbf{UV}\}\|_2^2 + \lambda \|\Psi_w\{\mathbf{UV}\}\|_1 \quad (1) \quad \text{and} \quad \hat{\mathbf{U}} = \arg\min \|\mathbf{d} - \Omega\{\mathbf{UV}\}\|_2^2 + \lambda \|\Psi_w\{\mathbf{UV} - \mathbf{C}_m\}\|_1 \quad (2)$$

where $\mathbf{d}$ denotes the acquired k-space data, Ω is the spatial Fourier transform followed by an undersampling operator, Ψ_w is the spasifying operator performing wavelet transform column by column, $\mathbf{C}_m = [\rho_{m1}, \rho_{m2}, ..., \rho_{mN}]$ is the isotropic signal whose each column is the average result across all directions: $\rho_{mi} = \frac{1}{N}\sum_{j=1,2,...,N} \rho_j$, λ is the regularization factor. Problem (1) and (2) are convex optimization problems and can be solved using additive half-quadratic regularization with continuation procedure [6].

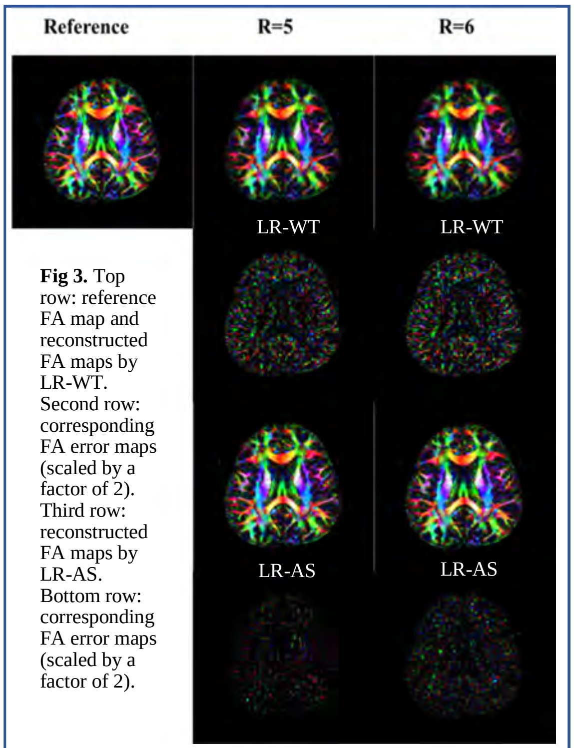

Fig 3. Top row: reference FA map and reconstructed FA maps by LR-WT. Second row: corresponding FA error maps (scaled by a factor of 2). Third row: reconstructed FA maps by LR-AS. Bottom row: corresponding FA error maps (scaled by a factor of 2).

Results: We have evaluated the proposed method based on the two sparsity constraints mentioned above using the same set of data. The in vivo brain DWI data using single-shot EPI were acquired on a Philips 3T scanner and then registered. The imaging parameters were: FOV=204mm×204mm, spatial resolution=2mm×2mm, TE/TR=70/8000ms, slice thickness=2mm, b-value=800s/mm^2, number of directions=32, reconstruction matrix size=128×128. The experimental data were generated artificially using a Poisson Disk sampling pattern with the center k-space of size 32×32 full sampled and total reduction factor R=3,4,5,6, respectively. Fig. 1 shows the root mean square error (RMSE) with model rank L. The proposed method based on LR-WT and LR-AS allows us to have a minimum RMSE at a rather low rank choice while the reconstruction using anisotropic sparsity reveals better quality. Fig. 2 shows the RMSE of each R at rank L=6, which reveals that the anisotropic sparsity especially induces robustness to high reduction factors. Fig. 3 compares the reference and reconstructed FA maps by LR-WT and LR-AS at R=5 and 6. As can be seen, the anisotropic sparsity provides better FA map reconstruction when R goes high. At lower R (3 or 4), both methods provide rather small and noise-like reconstruction errors.

Conclusion: In this work, we jointly enforce the low-rank and sparsity constraints for the reconstruction of DTI. We accelerate DTI acquisition by achieving high reduction factors of k-space. We show that the proposed method results in low RMSE at high reduction factors, and reconstruction based on anisotropic sparsity is more robust to a high undersampling pattern compared to traditional wavelet transform sparsity.

References: [1] Le Bihan et al. JMRI, 2001. [2] Assaf et al. JMN, 2008. [3] Gao et al. MRM, 2013. [4] Lam et al. IEEE ISBI, 2012. [5] Liang et al. IEEE ISBI, 2007. [6] Zhao et al. IEEE TMI, 2012. [7] Shi et al. MRM, 2014.