Consistent SNR Measures for Magnetic Resonance Elastography

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Target Audience: Physicians, scientists, and engineers interested in magnetic resonance elastography (MRE).

Purpose: In magnetic resonance elastography (MRE), the calculated stiffness values are affected by noise, which is amplified by the inversion process. It would be useful in practice to establish that, beyond some SNR threshold, stiffness calculations are accurate within some confidence limit. The most common methods to calculate SNR values in MRE estimate the noise in the measured displacement. However, the accuracy of stiffness determination depends not only on displacement SNR, but also on the wavelength of the shear wave, which is in turn dependent on the stiffness of the underlying material. More recently, octahedral shear strain (OSS) SNR has been proposed to be a more appropriate measure [1], since shear deformation is the signal in MRE. We demonstrate here that the proper SNR measure depends on the inversion algorithm used, and, more precisely, on the order of derivatives in the inversion process.

Methods: *Shear modulus calculation*: Dynamic MRE is based on harmonic mechanical excitation, and under common assumptions (isotropic material, local homogeneity, longitudinal waves removed via spatial filtering or the curl operator), the shear modulus can be calculated from the Helmholtz equation [2], $G = -\rho\omega2u/\Delta u$, with G the shear modulus, u the displacement and Δu its Laplacian, ω the angular frequency of the mechanical oscillation, and ρ the density of the material (usually assumed to be 1 for soft tissues). Inversions based on this principle of Helmholtz or direct inversion (DI) are perhaps the most commonly used in the MRE community, with many variations proposed. Alternative inversions [2] such as local frequency estimation (LFE) or phase gradient (PG) used for special purposes can be derived with additional assumptions (both of these can only calculate wave speed, and PG also assumes that only a single propagating wave is present). Other inversion approaches have been proposed as well, such as finite-element based non-linear inversion [1] and traveling wave expansion [3], but will not be considered here.

SNR calculation: MRE typically involves the collection of displacement data over a given number of phase offsets throughout a harmonic cycle. The 1st harmonic of the data (the motion at the driving frequency) is then obtained by fitting a sinusoid using the Discrete Fourier Transform (DFT). The noise in this quantity can be estimated from background areas or from the residuals in the fit. SNR can then be calculated by taking the ratio of the amplitude of the first harmonic over the estimated noise in this quantity. McGarry et al. proposed calculating the OSS SNR by calculating residuals as above and propagating them through the OSS formula [1]. They also stated that for the finite element subzone inversion technique used by their group, OSS SNR values greater than 3 led to stable, accurate inversions [1]. However, this does not ensure that OSS SNR is the proper measure for all inversion techniques. For DI, noise propagation calculations show that when the derivative kernel size is significantly smaller than the wavelength, the noise in the Laplacian of the data is the dominant term. We therefore also calculated the SNR of the Laplacian of the data, using a residual-based approach similar to above.

Simulated data: 3D images were created of simple plane sinusoidal waves at different wavelengths, with varying amounts of noise, simulating elastic materials of known stiffness (fig. 1). SNR and stiffness values were calculated for the simulated data. All derivative calculations were based on central differences with nearest neighbors, the smallest possible derivative kernel, with no filtering or smoothing of any sort. Although these simulations are extremely simple, *they suffice to capture key behavior that a consistent SNR measure must minimally satisfy.*

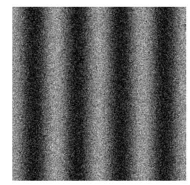

Fig. 1. Sample simulated data.

Results: Fig. 2(a) shows the mean stiffness (|G*|), calculated with DI and normalized to the correct value, for three simulations as a function of displacement SNR. The offsets between the curves indicate that an SNR value sufficient for accurate inversion for a softer object can yield very inaccurate results for a stiffer object. This is due to the derivative calculation in the inversion process: stiffer objects have longer wavelengths and thus smaller derivatives, while the noise level is unchanged. Fig. 2(b) shows that OSS SNR, which involves first derivatives, only partly improves this inconsistency. Fig. 2(c) shows that Laplacian SNR consistently predicts accuracy independent of stiffness. Similar results hold true for all quantities commonly calculated with DI – storage modulus G', loss modulus G", phase angle, or wave speed squared. The nature of the errors at low SNR (i.e. the shapes of the curves) differs among these, but the *consistency* of behavior with SNR across different stiffness values occurs only with Laplacian SNR. Standard deviation curves for all these quantities (not shown) also become consistent only with Laplacian SNR. Conversely, for LFE (which can be related to first derivative calculations) and for PG (which performs only first derivative calculations), OSS SNR does indeed predict accuracy and precision consistently independent of actual stiffness (data not shown). This appears to be true for non-linear finite element based inversion techniques as well [1]. If the curl of the data is taken in order to remove longitudinal waves, this involves an extra derivative. Results similar to fig. 2 show that this extra derivative must be accounted for - either by calculating SNR after the curl operation, or by using a higher-order derivative SNR measure. These results also highlight the high sensitivity to noise intrinsic to DI calculations, particularly if the curl is applied.

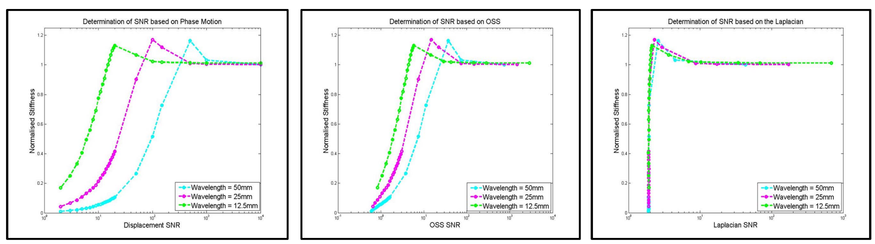

Fig. 2. Normalized stiffness as a function of displacement SNR, OSS SNR and Laplacian SNR.

Discussion: The results show that the appropriate SNR measure for MRE, which can consistently indicate the accuracy of the stiffness estimation, depends on the inversion algorithm that is used, and more precisely on the order of derivatives involved. Nearest neighbor derivatives were used here for clarity. With actual data, smoothing or use of larger derivative kernels prior to or concurrent with the stiffness calculation is usually required. This shifts the curves horizontally (even if SNR is calculated after smoothing, due to correlated noise), but *it does not alter the consistency* (or lack thereof) of the SNR measures. Inversion techniques involving weak variational formulations ([4-5]) can be shown to be numerically equivalent to smoothing, and the consistency results hold true for these approaches as well.

Conclusion: The proper SNR measure for MRE depends on the inversion algorithm used, and, more precisely, on the order of derivatives in the inversion process. Laplacian-based SNR measures are required to consistently analyze commonly used Helmholtz inversions.

References: [1] McGarry et al., Phys Med Bio 56:N153:N164, 2011. [2] Manduca et al., Med Img Anal 5:237-254, 2001. [3] Baghani et al., IEEE TMI 30:1555-1565. [4] Romano et al., IEEE UFFC 45:751-759. [5] Cortes et al., MRM 72:211-219.