Sparsity-Promoting Orthogonal Dictionary Updating for Highly Undersampled MRI Reconstruction

Jinhong Huang[1,2], Xiaohui Liu[1], Wufan Chen[1], and Yanqiu Feng[1]

[1]*Guangdong Provincial Key Laboratory of Medical Image Processing, School of Biomedical Engineering, Southern Medical University, Guangzhou, Guangdong, China,*
[2]*School of Mathemtics and Computer Science, Gannan Normal University, Ganzhou, Jiangxi, China*

Introduction: Exploiting redundancy among image patches by using dictionary learning methods has demonstrated superior performance than using predefined analytical bases in compressed sensing MRI[1,2]. However, the synthesis sparse coding problem in the K-SVD dictionary learning algorithm[3] is NP-hard and thus the K-SVD based dictionary learning MRI (DLMRI)[2] is computationally expensive. Instead of learning overcomplete dictionary, in this work, we introduce a novel sparsity-promoting orthogonal dictionary updating (SPODU) method for more efficient image reconstruction from highly undersampled MRI data. To further improve reconstruction accuracy, the sparsity level is gradually increased during iterations for recovering more details, and the contribution of sparse representation using updated dictionary is gradually increased based on progressively improved quality of dictionary.

Methods: Given the vectorized version of incoherently undersampled k-space measurements data $\mathbf{y}$, the purpose of sparse MRI reconstruction is to obtain an accurate estimate of underlying image $\mathbf{x}$ from $\mathbf{y}$ through sparsity-promoting algorithms. Formally, the proposed SPODU method can be stated as: $\min_{\mathbf{x},D,\Gamma} \sum_i \|R_i\mathbf{x} - D\boldsymbol{\alpha}_i\|_2^2 + \lambda\sum_i \|\boldsymbol{\alpha}_i\|_0 + \nu\|F_u\mathbf{x}-\mathbf{y}\|_2^2$, $s.t.\ D^H D = I_n$. Therein, F_u is the undersampled Fourier encoding matrix , R_i denotes the operator which extracts a patch of size n from image $\mathbf{x}$, D represents the dictionary consisting of K columns referred to as atoms, $\boldsymbol{\alpha}_i$ denotes the representation coefficient vector of $R_i\mathbf{x}$ with respect to D, and Γ denotes the sparse representation of all patches in the image.

The above minimization problem can be solved by a procedure which alternatively updates one of the three variables (Γ, D and $\mathbf{x}$) while keeps the other two fixed. Initially, the image $\mathbf{x}^{(0)}$ is obtained by zero-filled Fourier reconstruction. Then, the image patch matrix X, which the *i*-th column is $R_i\mathbf{x}$, can be initialized by extracting all patches form $\mathbf{x}^{(0)}$, and the orthogonal dictionary $D^{(0)}$ is determined as the left singular vectors of a matrix whose columns consist of a fraction of columns from $X^{(0)}$. Given fixed D and X, the solution $\Gamma*$ can be uniquely and exactly obtained by applying an element-wise hard thresholding operator[4] to $D^H X$ .Given fixed X and Γ and suppose that $X\Gamma^H = U\Sigma V^H$ is abitrary singular value decomposition (SVD) of $X\Gamma^H$, then $D^* = UV^H$ is the optimal solution to update the orthogonal dictionary. With fixed Γ and D, the reconstruction $\mathbf{x}$ can be updated by minimizing a least squares problem under the 'wrap around' assumption [2], Thus, the subproblem in each step can be efficiently solved with an closed form solution.

Algorithm 1. The procedure of the SPODU algorithm

Input：k-space measurements $\mathbf{y}$

Output：Reconstructed image $\mathbf{x}$

Initialization：λ_0, ν_0, $\mathbf{x}^{(0)} = F_u^{\ H}\mathbf{y}$, $D^{(0)}$ is obtained by SVD method

Iteration: For the *k*-th iteration

1) Set $\lambda = \lambda_0 \cdot \delta^k$ and $\nu = \nu_0 \cdot \delta^k$
2) Exacting patches: $X^{(k-1)} = [R_1\mathbf{x}^{(k-1)}, R_2\mathbf{x}^{(k-1)}, \cdots, R_N\mathbf{x}^{(k-1)}]$
3) Sparse coding: $\Gamma^{(k)} = H_{\sqrt{\lambda}}\left((D^{(k-1)})^T X^{(k-1)}\right)$
4) Orthogonal dictionary updating: $D^{(k)} = UV^H$
5) Updating reconstruction via solving a least squares problem: $\mathbf{x}^{(k)} = \arg\min_{\mathbf{x}} \sum_i \|R_i\mathbf{x} - D\boldsymbol{\alpha}_i\|_2^2 + \nu\|F_u\mathbf{x}-\mathbf{y}\|_2^2$

To further improve the reconstruction, the deterministic annealing[5] like strategy is combined into the above algorithm. Specifically, we gradually decrease the parameters λ and ν during iterations: $\lambda = \lambda_0 \cdot \delta^k$ and $\nu = \nu_0 \cdot \delta^k$, where *k* indicates the *k*-th iteration, λ_0 and ν_0 are the initial values, and δ is the decreasing factor. The complete procedure of the SPODU algorithm is summarized in Algorithm 1.

Results: Experiments were perfomred on phantom data[1] with retrospective 1D random Cartesian undersampling along phase encoding direction and 2D random undersampling schemes to evaluate the performance of the proposed SPODU method with a comparison to the DLMRI method.

Fig. 1 shows the comparison of phantom images reconstructed by DLMRI and SPODU from 4-fold 1D undersampled Cartesian data. From the enlarged view in Figs. 1(b) and 1(c), the SPODU reconstruction has clearer definition of edges than the DLMRI reconstruction. Similar results can be observed in Fig. 2, which presents the comparison of DLMRI and SPODU under 10-fold 2D random undersampling. SPODU offers more accurate reconstruction than DLMRI, as depicted in Fig. 2(b) and 2(c) by the better preservation of detailed structures. Quantitative improvement in terms of PSNR of SPODU over DLMRI can also be observed (Figs. 1(d) and 2(d)). It can be easily observed in Figs. 1(e) and 2(e) that the computational time for SPODU is dramatically lower than that of DLMRI. In total, SPODU is approximately one magnitude and a half (50x) faster than the K-SVD based DLMRI.

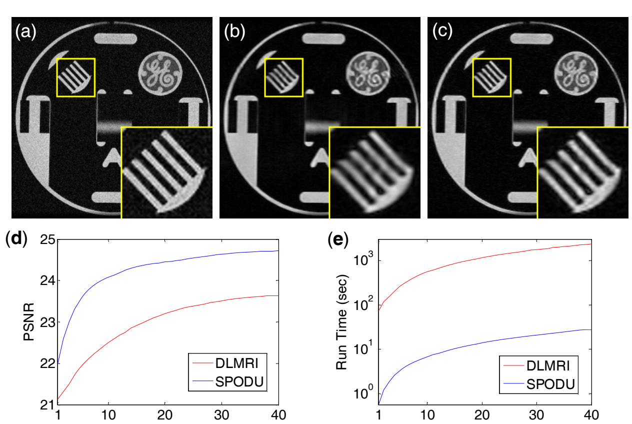

Fig. 1. Cartesian sampling. (a) Reconstruction with full sampled k-space data. (b) Reconstruction using DLMRI. (c) Reconstruction using SPODU. (d) PSNR versus iteration number. (e) Run time versus iteration number.

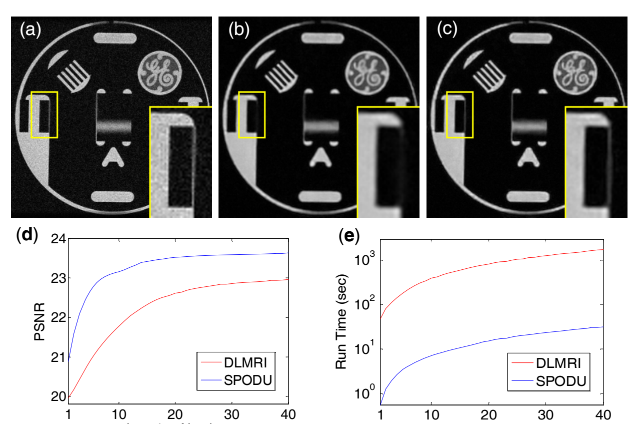

Fig. 2. 2D random sampling. (a) Reconstruction with full sampled k-space data. (b) Reconstruction using DLMRI. (c) Reconstruction using SPODU. (d) PSNR versus iteration number. (e) Run time versus iteration number.

Conclusions: Experimental results demonstrate that the proposed SPODU algorithm is more efficient and accurate than the DLMRI algorithm, and thus has potential application in practice.The combination of the proposed method with parallel MRI techniques, and its extension to dynamic MRI and diffusion MRI will be further investigated in our future research.

References: [1] Lustig et al., MRM, 2007; 58(6):1182-95. [2] Ravishankar et al., IEEE TMI, 2011; 35(5):1028-41. [3] Aharon et al., IEEE TSP, 2006; 54(11):4311-22. [4] Blumensath et al., J. Fourier Anal. Appl., 2008;14(5-6): 629-654. [5] Li, IEEE J-STSP, 2011; 5(5): 953-962.