Image Reconstruction of Under-sampled Signal at Equal Interval using Quadratic Phase Scrambling

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Introduction: Sparse MRI has been introduced to reduce the acquisition time and raw data size by randomly undersampling the k-space data. However, the image quality depends on the randomness or sampling density in k-space even if the reduction factor of signal is the same. In other words, the best signal trajectory depends on the object to be imaged, however, it is impossible in general to know the best signal trajectory for unknown image data. In this paper, we propose a novel image reconstruction technique in which undersampled signal at equal interval is adopted in image reconstruction. To reduce the aliasing artifact due to equally-spaced undersampled signal, we used the phase-scrambler in the acquisition.

Theory: To realize the reconstruction using the undersampled signal at equal interval, we introduced the phase-scrambling Fourier transform imaging (PSFT)[1] in which a quadratic field gradient for the phase scrambling is added to the pulse sequence of conventional FT imaging in synchronization with the field gradient for phase encoding.

$$v(k_x,k_y)=\int\int_{-\infty}^{\infty}\rho(x,y)e^{-j\gamma b\tau\left(x^2+y^2\right)}e^{-j\left(k_x x+k_y y\right)}dxdy \quad \cdots \ (1)$$

$$\rho(x,y)=\left(\frac{1}{2\pi}\right)^2 e^{j\gamma b\tau\left(x^2+y^2\right)}\int\int_{-\infty}^{\infty}v(k_x,k_y)e^{j\left(k_x x+k_y y\right)}dk_x dk_y \quad \cdots \ (2)$$

where $\rho(x,y)$ represents the spin density distribution in the subject, γ is the gyromagnetic ratio, and b and τ are the coefficient and impressing time, respectively, of the quadratic field gradient. Image reconstruction is executed by inverse Fourier transform followed by quadratic phase demodulating as shown in Eq.(2). The coefficient of phase scrambling $\gamma b\tau$ is normalized as $\gamma b\tau = h\gamma b\tau'$, where phase changes with neighboring pixel become π at $\gamma b\tau'=\pi/(N\Delta x^2)$ (N: size of image, Δx: pixel size).

Iterative Image reconstruction using undersampled PSFT signal at equal interval based on the Gerchberg algorithm have been presented using 50% of signal[2,3]. Phase scrambling applied to the spin density in Eq.(1) spread the reconstruction error to the k-space and the part of the error will be removed by replacing the signal with true one on the trajectory where signal was acquired in the iterative reconstruction. Therefore, the image quality strongly depends on the phase scrambling coefficient $\gamma b\tau$. Since phase scrambling applied to the object promotes the mutual incoherence between the sampling matrix and sparsifying function, it was demonstrated that the quality of reconstructed images will be improved when it is applied to compressed sensing (CS)[4]. In this work, we propose a new image reconstruction scheme to improve the reduction factor of our previous method and to reduce the dependency of sampling trajectory for compressed sensing. Combination of L1 nor minimization and the use of phase scrambling was considered to improve the practical use of sub-Nyquist reconstruction method.

Results and Discussion: We used the SpaRSA [5] algorithm to reconstruct images. To accelerate the convergence, FREBAS transform[6] was adopted as a sparsifying function and soft-thresholding is applied to minimizing the L1 norm in the FREBAS domain. PSFT signal is calculated using the MR volunteer image data according to the Eq.(1) in the simulation experiments,. We used Cartesian sampling because it is widely used in practice. The signal for the phase encoding direction is undersampled at equal interval in proposed method. Random sampling and variable density random sampling at best condition (PSFT-CS) were also applied for CS. Figure 1 (a), (b) show the averaged PSNRs of 10 images with reference to the phase scrambling coefficient h for proposed method and PSFT-CS using 33% and 25% signal, respectively. It was shown that highest PSNRs are obtained at different $\gamma b\tau$ in proposed method and PSFT-CS. Figure 2 shows the averaged PSNRs with reference to the signal reduction factor. Proposed method shows higher PSNR than PSFT random sampling and has almost the same PSNR with PSFT-CS. Figure 3 shows the comparison of obtained images using 33% and 25% of signal. Figs.(b) and (d) are the images obtained in proposed method and (c) and (e) are the images in PSFT-CS. Figure 4 shows the results of application to experimentally obtained data using 0.2T MRI scanner. Imaging parameters are as follows; N=256, spatial resolution Δx=0.08 cm, h=0.6. Fig.2, Fig.3 and Fig.4 show that reconstructed images obtained in proposed method are comparable to those of obtained in PSFT-CS with variable density sampling at best condition.

Conclusion: A new fast image reconstruction method that allows undersampling at equal interval is proposed and demonstrated. Almost the same quality of images were obtained in the sense of PSNR and visual inspection. Since the randomness of undersampling is not required in proposed method, the quality of reconstructed images do not depend on the sampling trajectory which will improve the practical utility of the method.

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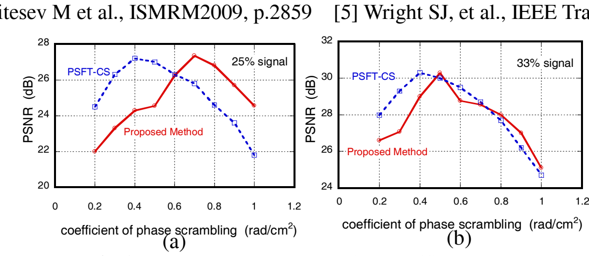

Fig.1 PSNR vs phase scrambling coefficient for 33% and 25% signal.

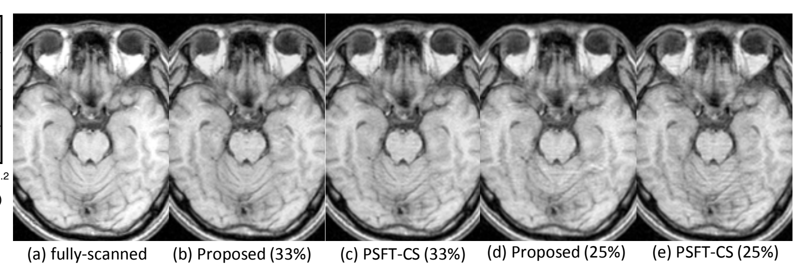

Fig.3 Comparison of reconstructed images with PSFT-CS

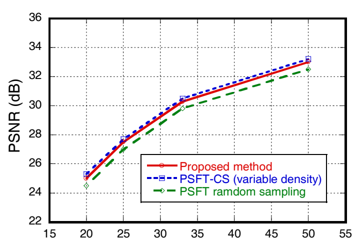

Fig.2 PSNR vs phase scrambling coefficient

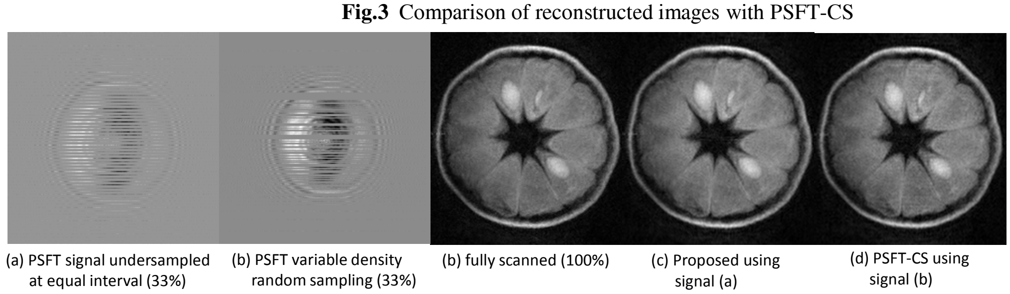

Fig.4 Application of proposed method to experimentally obtained signal (h=0.6)