Analytic form 3D radial sampling strategy for maintaing the uniformity of k-space coverage with increasing interleaves

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Target Audience: MR physicists and radiologists

Purpose: In 3D radial-acquisition(RA) imaging, one of the most popular ways fulfilling a nearly uniform *k*-space coverage is to distribute the coordinates of all projections (or views) along the spiral trajectory running on a sphere from one pole to the other or to the equator, based on a simple analytic expression suggested by Wong et.al (1,2). Although Wong's method provides a nearly uniform *k*-space coverage with no interleaves or a few interleaves, it fails to maintain the *k*-space uniformity as the number of interleaves(i_{max}) increases, which may cause a decrease of SNR (3) as well as image artifacts such as blurring and streak artifacts. In this study, we propose a modified version of Wong's method that is able to almost maintain the uniformity of *k*-space coverage even in the event that i_{max} increases, providing an analytic form for the coordinates of all views in terms of the readout gradient strengths in the *x*-, *y*-, and *z*- axes. Our suggestion was validated by simulation and phantom experiments.

Methods: Theory: According to Wong et al., the equations of a spiral trajectory for the normalized coordinates of *x*-, *y*-, and *z*- readout gradients are presented by:

$$G_z(p) = \frac{2p - p_{max} - 1}{p_{max}},\ G_x(p,i) = \cos\left(\sqrt{\frac{p_{max}\pi}{i_{max}}}\sin^{-1}(G_z(p)) + \frac{2i\pi}{i_{max}}\right)\sqrt{1-(G_z(p))^2},\ G_y(p,i) = \sin\left(\sqrt{\frac{p_{max}\pi}{i_{max}}}\sin^{-1}(G_z(p)) + \frac{2i\pi}{i_{max}}\right)\sqrt{1-(G_z(p))^2},[1]$$

where p is the p^{th} view of an interleaf, p_{max} is # of view per interleaf, and i is the i^{th} interleaf. Eq.[1] provides a nearly uniform *k*-space coverage when $i_{max} = 1$. However, as i_{max} increases, non-uniformity in *k*-space begins to appear, especially along the polar direction because the distance between two adjacent points along the *z*-direction linearly increases with i_{max} (Fig.1a). To overcome the weakness of Eq.[1], we present a modified version of Eq.[1] that can almost maintain the uniformity of *k*-space coverage as i_{max} increases like the following:

$$G_z(p,i) = \frac{2((p-1)i_{max} + i) - N_{views} - 1}{N_{views}},\ G_x(p,i) = \cos\left(\sqrt{N_{views}\pi}\cdot\sin^{-1}(G_z(p,i))\right)\cdot\sqrt{1-(G_z(p,i))^2},\ G_y(p,i) = \sin\left(\sqrt{N_{views}\pi}\cdot\sin^{-1}(G_z(p,i))\right)\cdot\sqrt{1-(G_z(p,i))^2},[2]$$

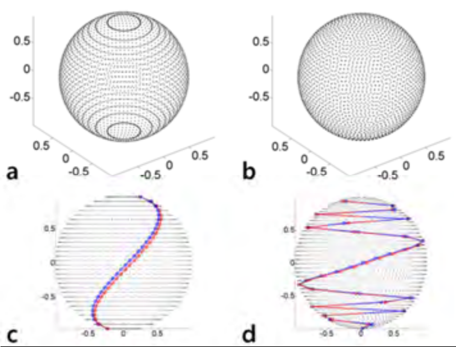

Fig.1 3D *k*-space spiral trajectories. (a) Trajectory generated by Eq.[1] and (b) by Eq.[2]. N_{views} = 3 k, i_{max} = 100. (c) and (d) illustrate how Eqs.[1] and [2] generate the interleaves from a 2D perspective viewed at the frontal side, respectively, representatively showing 1st interleaf(blue) and 2nd interleaf(red).

where N_{views} is the total number of views (= $p_{max}\cdot i_{max}$). The underlying idea of Eq.[4] is that each interleaf is generated from the one-path spiral trajectory itself that has i_{max} = 1 and same N_{views}, by assigning every i_{max} view to a specific interleaf (Fig.1d). Simulation: To show the uniformity of *k*-space coverage, 3D *k*-space trajectories were demonstrated using Eqs.[1] and [2]. For better representation, N_{views} was set to be 3k with i_{max} = 1 and 100, respectively. To quantitatively evaluate the uniformity as N_{views} varies with a fixed i_{max} in Eq.[1], we applied an imaginary circle to each point, measuring the number of points inside the circle(N_{circle}), and calculated the standard deviation(STD) of the N_{circle} distribution. The diameter of the imaginary circle was scaled by the distance between two adjacent points as N_{views} changes. i_{max} was assumed to be 100 and N_{views} varied from 3k (p_{max}=30) to 500k (p_{max}=500). Experiment: For experimental validation, ACR phantom imaging was performed at 3T (Siemens Magnetom Trio, Erlangen, Germany) with a volume coil. For 3D RA, a recently proposed gradient-echo-based UTE sequence, CODE(Concurrent Dephasing and Excitation), was used (4). Scan parameters were as follows: TE/TR = 0.14 ms/3 ms, FOV = 300 mm, FA = 5°, BW = 530 Hz/pixel, N_{views} = 3/30/100 k, i_{max} = 100, and scan time = 9/90/300 s. Image was reconstructed offline with a home-built MATLAB (Mathworks, R2011a) program using gridding algorithm.

Results and Discussion: Simulation: As demonstrated in Figs 1a and b, Eq.[1] produced non-uniform *k*-space coverage with i_{max} = 100, whereas Eq.[2] maintained the uniformity of *k*-space coverage. Figures 1c and d visually show how Eqs.[1] and [2] generate a spiral trajectories with interleaves. According to Fig.2, the STD of the N_{circle} distribution was reduced when N_{views} increased, which implies that, with same i_{max}, the uniformity of *k*-space coverage becomes better as N_{views} increases when using the Wong's method (Eq.[1]). Experiment: Figures 3a, b, and c show the phantom images when Eq.[1] was used to generate the *k*-space trajectories with N_{views} = 3k, 30k, and 100k, respectively. Figures 3d, e, and f are the images obtained when Eq.[2] was used. As expected, Figs 3a, b, and c show under-sampling artifacts along the *z* direction (white arrows), which diminishes as N_{views} changes from 3k (a) to 100k (c), as expected from Fig.2. In contrast, these artifacts are barely seen in Figs 3d, e, and f due to the maintained uniformity of *k*-space coverage. Overall blurring seen in (a) and (b) is attributed to the use of a very small number of N_{views} (= 3k) to amplify the non-uniformity.

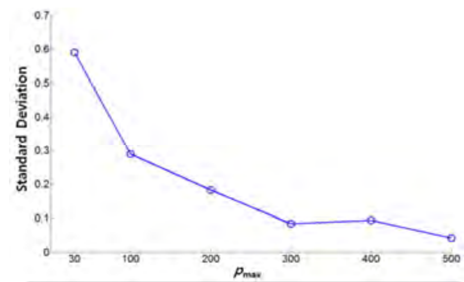

Fig.2 Plots of the STD of N_{circle} distribution vs. p_{max}. i_{max} = 100 and N_{views} varied from 3k (p_{max}=30) to 500k (p_{max}=500).

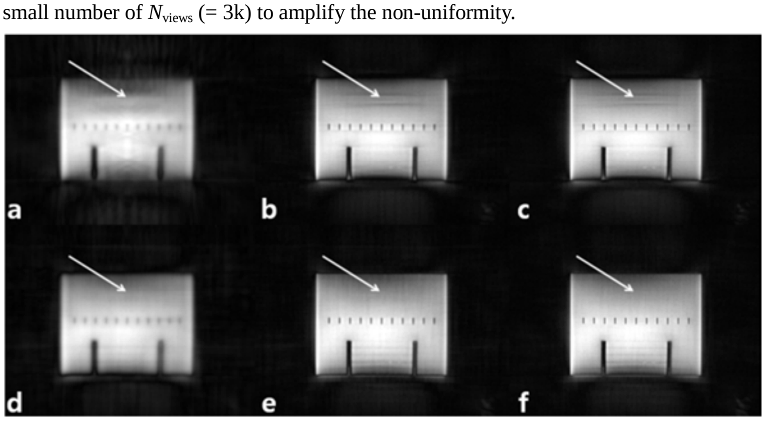

Fig.3 Sagittal slices of ACR phantom images. (a) and (d) were obtained with N_{views} = 3k, (b) and (e) with N_{views} = 30k, (c) and (f) with N_{views} = 100k. i_{max}=100. Images in the first-row were obtained using Eq.[1] and images in the second row using Eq.[2].

Conclusion: Here we proposed a modified version (Eq.[2]) of Wong's method that can maintain the uniformity of *k*-space coverage even in the case of increasing the number of interleaves, providing a mathematical expression for the readout gradient strengths in the *x*-, *y*-, and *z*- axes. Simulation and phantom experiment results well confirmed the performance of our new method. We also showed that the non-uniformity of *k*-space coverage decreases as the total number of views increases when using the Wong's method (Eq.[1]).

Reference: [1]Wong ST, Roos MS. A strategy for sampling on a sphere applied to 3Dselective RF pulse design. MagnReson Med 1994;32:778–784.[2]Stehning C, Bornert P, Nehrke K, et al., Fast Isotropic Volumetric Coronary MR Angiography Using Free-Breathing 3D Radial Balanced FFE Acquisition. MagnReson Med 2004;52:197-203. [3]Jan-Ray L,Pauly JM, Pelc NJ, et al. "Reduction of motion artifacts in cine MRI using variable-density spiral trajectories." MagnReson Med1997;37.4: 569-575.[4] Park JY, Moeller S, Goerke U, et al.,Short Echo-Time 3D Radial Gradient-Echo MRI Using Concurrent Dephasing and Excitation. Mag Reson Med 2012;67:428-436. **Acknowledgment**: This work was supported by IBS-R015-D1-2014-a00 and NRF 2010-0025744.