CODEC: Covariance-driven Parallel Imaging for NonCartesian Sampling Trajectories

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INTRODUCTION: The common parallel imaging methods SENSE(1) and GRAPPA(2) are well-suited for uniform undersampling, but become more computationally burdened when modified for nonCartesian sampling (particularly 3D). The proposed parallel imaging method, CODEC (Convolution Operations for Data Estimation from Covariance), is a k-space convolution method that makes no assumption of sampling geometry, and does not require training.

THEORY: For estimating a value e_i at a k-space location for some coil, using data d_j measured at D locations from N coils, first define $s = \mathbf{a}^T\mathbf{d}$, where **a** and **d** are vectors (length ND) of weights and measured data. The conditional mean of e_i based on s is given as a function of their correlation ρ_{es}, as shown in Eq. [1], where σ_e and σ_s are the standard deviations of e and s. The error of Eq. [1] is minimized by choosing **a** via Eq. [2], where $\mathbf{r_{ed}}$ is a vector of covariances between e_i and each element of **d**, and $\mathbf{C_{dd}}$ is the NDxND covariance matrix for all pairs of the elements of **d**. With Eqs. [1] and [2], the vector **e** of all estimated values e_i is then obtained with Eq. [3], where $\mathbf{C_{ed}}$ is a matrix of covariances between each pair of e_i and d_j. Note Eq. [3] is also presented in the APPEAR method(3), but is derived and used very differently. The proposed solution to Eq. [3] is a 2-step algorithm. First, one uses (e.g.) a Conjugate Gradient (CG) method to solve for $\delta = \mathbf{C}^{-1}_{\mathbf{dd}}\,\mathbf{d}$ via Eq. [4]. Once δ is estimated, Eq. [3] is easily solved as Eq. [5].

$$E[e_i\,|\,s] = \rho_{es}\; s\left(\sigma_e/\sigma_s\right) \quad [1]$$

$$\mathbf{a} = \sigma_e \mathbf{C}^{-1}_{\mathbf{dd}}\mathbf{r}_{\mathbf{ed}} \Big/ \sqrt{\mathbf{r}^{\mathbf{H}}_{\mathbf{ed}}\mathbf{C}^{-1}_{\mathbf{dd}}\mathbf{r}_{\mathbf{ed}}} \quad [2]$$

$$\mathbf{e} = \mathbf{C}_{\mathbf{ed}}\mathbf{C}^{-1}_{\mathbf{dd}}\,\mathbf{d} \quad [3]$$

STEP 1: $$\mathbf{d} = \mathbf{C}_{\mathbf{dd}}\delta \quad [4]$$

STEP 2: $$\mathbf{e} = \mathbf{C}_{\mathbf{ed}}\delta \quad [5]$$

METHODS: In practice, **C** matrices are not used; rather, Eqs. [4,5] are solved by convolving with kernels defining data covariance as a function of relative distance dk in k-space for each coil pair. These kernels are estimated by first taking the product $f_m\, f^*_n$ of low resolution (fully sampled) images f from each coil pair (m,n) (Fig. 1a). The Fourier Transform of each product gives an estimate of the data covariance between coils m (row) and n (column) as a function of dk (Fig. 1b). Data were collected on a Philips 3T Ingenia using both 2D spiral (240 mtx) and 3D Spherically Distributed Spirals (165^2 x 122 matrix) trajectories(4), each fully sampled for $|k_{xy}| < 0.1$/FOV, with increasing undersampling to R=3 for $|k_{xy}| \geq 0.3$/FOV.

RESULTS: Computation time for Eq. [4] was 10.5 sec (16 iterations) for 2D [24 arms, 1115 pts/arm, 5 coils, kernel radius 4/FOV] and 117 min (16 iterations) for 3D [973 arms, 1282 pts/arm, 5 coils, kernel radius (4x4x2)/FOV], for a single thread on a 2.6GHz Macbook Pro using C code in GPI software(5). Example images are shown in Fig. 3.

DISCUSSION: CODEC is a fast, flexible method for parallel imaging, and requires only convolutions during the CG process - but needs N^2 (for N coils) of them. Reducing N with (e.g.) Virtual Coil compression(6) is thus beneficial for computation times. Parallelization, optimization, comparisons to come.

REFERENCES: 1. Pruessmann et. al, MRM 42(5): 952-62. 2. Griswold et. al, MRM 47(6): 1202-10. 3. Beatty et. al, ISMRM 2007, abs # 335. 4. Turley, et. al, MRM 70(2): 413-9. 5. Zwart, et. al, MRM, DOI 10.1002/mrm.25528. 6. Buehrer et. al, MRM 57(6): 1131-39. **ACKNOWLEDGEMENTS:** This work was funded by Philips Healthcare. The author gratefully thanks Dinghui Wang for providing data.

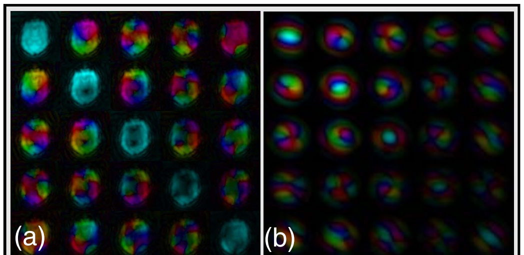


Fig. 1. Row m, column n shows (a) coil image products $f_m\, f^*_n$ and (b) their FT, i.e. covariance as a function of dk. Color indicates phase.

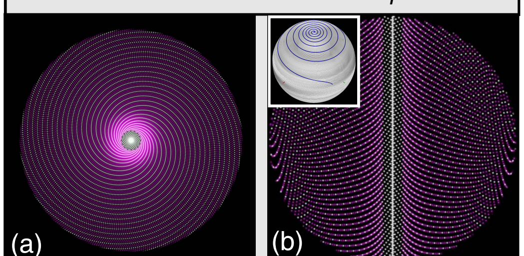
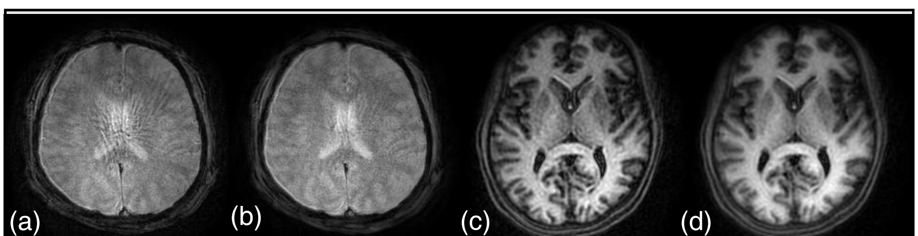


*Fig. 2. Example sampling patterns for (a) 2D and (b) k_x-k_z plane of 3D spherically distributed spiral trajectories (inset = 3D view). Measured data (white = **d**) were used in the fully sampled center to make covariance maps, and to estimate data (magenta = **e**) elsewhere.*



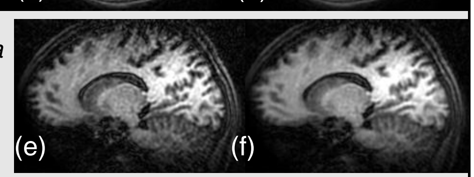


Fig. 3. Reconstructed images (before deblurring) for (a,c,e) undersampled data (R=3) and (b,d,f) data with CODEC estimates for (a,b) 2D and (c-f) 3D spherically distributed spiral data. 3D aliasing (c,e) appears as clutter.