Parallel MRI Reconstruction by Direct Convex Optimization

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Purpose: The purpose of the proposed direct convex optimization approach to parallel magnetic resonance imaging (pMRI) is to provide a computational algorithm for efficient processing of undersampled k-space data and accurate reconstruction of magnetic resonance images. The pMRI is implemented with multiple coils and an undersampling scheme for speeding up the scan process. The image reconstruction for pMRI is formulated into a regularized optimization problem in a number of recent research results. Because of the coupling between the image and coil sensitivity functions, the optimization solution for the image function, subject to unknown coil sensitivity functions, is in general a non-convex problem. To solve such a non-convex problem involves difficulties in computational complexity and local only solutions. Our proposed approach is based on an analysis that the magnitude image function to be solved is contained in a convex hull in the solution space formed by the magnitude image and the sensitivity encoded image functions. This convex hull enables a novel formulation of the pMRI reconstruction with unknown sensitivity functions into a direct convex optimization problem for efficient and accurate pMRI reconstruction.

Theory: The undersampled k-space data of a pMRI scanner with L receiver coils can be written as $\mathbf{g}_i = \mathbf{F}(\mathbf{s}_i \bullet \mathbf{h}) = \mathbf{F}\mathbf{z}_i,\ i = 1...L,$ where $\bullet$ denotes the pointwise product between vectors, $\mathbf{g}_i \in \mathbb{C}^M$, $\mathbf{s}_i \in \mathbb{C}^{N^2}$ and $\mathbf{z}_i = \mathbf{s}_i \bullet \mathbf{h} \in \mathbb{C}^{N^2}$, with $M < N^2$, are the undersampled k-space data vector, the vector of the sensitivity function and the vector of the sensitivity encoded image of the *i*th coil, respectively, and $\mathbf{h} \in \mathbb{C}^{N^2}$ is the vector of the image function. Since the inductance values of the receiver coils are bounded, the coil sensitivity functions are bounded. Thus there exist constant vectors $\mathbf{b}_i \in \mathbb{R}_+^{N^2}$ such that the magnitude of $\mathbf{s}_i$ is elementwisely bounded by $\mathbf{b}_i$ denoted by $|\mathbf{s}_i| \le \mathbf{b}_i$. It follows that $|\mathbf{z}_i| \le \mathbf{b}_i \bullet \mathbf{h}_m,\ i = 1...L,$ where $\mathbf{h}_m = |\mathbf{h}|$ is the magnitude vector of $\mathbf{h}$.

In the $2N^2$ dimensional coordinate system formed by $\{|\mathbf{z}_i|, \mathbf{h}_m\}$, the vector inequality $|\mathbf{z}_i| \le \mathbf{b}_i \bullet \mathbf{h}_m$ defines a convex hull. With a properly chosen value of $\mathbf{b}_i$ based on the *a priori* knowledge of the receiver coil and the scanning condition, the convex hull can contain the true values of $|\mathbf{z}_i|$ and $\mathbf{h}_m$. This analysis can enable the following formulation of the l_1 regularized convex optimization problem for solving the magnitude image vector $\mathbf{h}_m$.

$$\min_{\mathbf{z}_i, \mathbf{h}_m,\ i=1...L} \frac{1}{2}\sum_{i=1}^{L} \| \mathbf{g}_i - \mathbf{F}\mathbf{z}_i \|_2^2 + \lambda \| \mathbf{W}\mathbf{h}_m \|_1, \ \text{subject to } \mathbf{h}_m \ge 0,\ \ |\mathbf{z}_i| \le \mathbf{b}_i \bullet \mathbf{h}_m,\ i = 1...L,$$

where $\mathbf{W}$ is the Haar wavelet transform matrix for sparsifying the magnitude image and λ is a regularization parameter.

Methods and Results: The above formulation is implemented with the split Bregman algorithm and applied to a brain data set of dimension 256x256 acquired from a 3T SIEMENS Trio scanner with an 8-channel head array and an MPRAGE sequence. The fully acquired data set in the Cartesian coordinate system are manually undersampled at different acceleration rates, respectively, with additional 36 extra auto-calibration lines in the central region along the phase encoding direction. In the computation of each reconstruction, the initial image pixel values are set randomly and the regularization parameters and the upper bound vectors $\mathbf{b}_i$ are experimentally selected. To evaluate the reconstruction accuracy, the reconstructed magnitude image $\mathbf{h}_m$ is compared with the sum-of-square image $\mathbf{h}_{sos}$ of the full data set in terms of the normalized mean squares error which is defined as $e_{NMSE} = \| \mathbf{h}_m - \mathbf{h}_{sos} \|^2 / \| \mathbf{h}_{sos} \|^2$.

The brain data sets at undersampling rates r=4,8,12,16 are reconstructed by the proposed algorithm as well as by state of the art algorithms for performance comparison. These algorithms are the Nonlinear reconstruction using variational penalties (IRGN), Iterative self-consistent pMRI (SPIRiT), Eigenvalue approach to autocalibrating pMRI (ESPIiT), Joint image reconstruction and sensitivity estimation in SENSE (JSENSE) and GRAPPA algorithms. The e_{NMSE} values of the reconstructed images are obtained and recorded in Table 1. In Fig.1, the reconstructed brain images at r=8 with zoomed parts by different algorithms are shown. The reconstruction results can also demonstrate the computational advantages and the uniqueness of the reconstruction solution under different initial image conditions of our algorithm.

r	Our Method	IRGN	CG-SPIRiT	ESPIRiT	GRAPPA	JSENSE
4	0.0024	0.0036	0.0032	0.0029	0.0064	0.0052
8	0.0032	0.0048	0.0049	0.0045	0.0102	0.0072
12	0.0043	0.0065	0.0068	0.0066	0.0125	0.0096
16	0.0058	0.0092	0.0110	0.0097	0.0217	0.0128

Table 1. The e_{NMSE} values of the reconstructed images different algorithms at different undersampling rates.

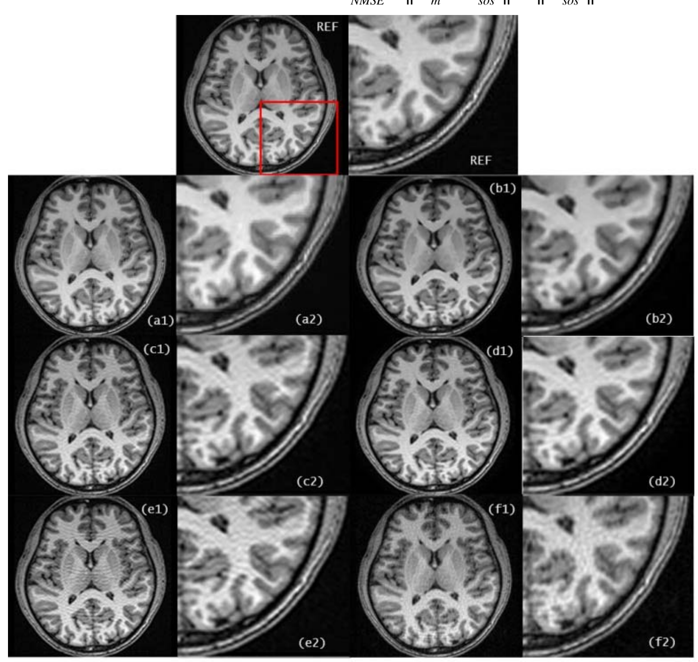

Fig.1. Reconstructed brain images by different algorithms at undersampling rate r=8. REF: the SOS image and a zoomed part; Form (a1, a2) to (f1, f2) are, respectively, the reconstructed images and the corresponding zoomed parts by our proposed algorithm, IRGN, CG-SPIRiT, ESPIRiT, GRAPPA and JSENSE algorithms.

Conclusion: The proposed direct convex optimization method overcomes the non-convex difficulties of the pMRI reconstruction problem and can provide accurate and computationally efficient reconstruction solutions. Its reconstruction performance has been demonstrated in comparison with state of the art pMRI reconstruction algorithms.