Iterative GRAPPA using Wiener filter

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Purpose: Since GRAPPA [1], several iterative GRAPPA approaches have been proposed [2-4]. They use arbitrary k-space sampling [2], or iteratively estimate more accurate convolution weights using regularization [3] or Kalman filtering on weights [4]. In this paper, we present a new iterative method using Wiener filter. Conventional GRAPPA method uses the auto calibration signals (ACS) to find the convolution weights. We however note that the convolution weights obtained from ACS cannot represent the relationship between the acquired and missing data accurately. To address this issue, we propose a method that iteratively updates the convolution weights using both the acquired and reconstructed data from previous iterations in the entire k-space. To avoid error propagation, the method applies adaptive Wiener filter on the reconstructed data where the power spectra are estimated from the neighboring data adaptively without any prior information. The method shows improvement in SNR over GRAPPA.

Method: We use the multicolumn multiline interpolation (MCMLI) with floating net based fitting (FNF) [5] for GRAPPA. At iteration index n=0, we use ACS lines to get the initial weights $w^{(0)}$ and set the initial filtered signal $\tilde{s}^{(0)}(k_y, k_x; j) = 0$. We then iterate the following three steps. (Step 1) We update the iteration index $n \rightarrow n+1$ and use $w^{(n-1)}$ and the acquired data to reconstruct the missing lines as follows:

$$\hat{s}^{(n)}(k_y, k_x; j)|_{k_y=kR+r} = \sum_{l=1}^{L} \sum_{b=-B_1}^{B_2} \sum_{h=-H_1}^{H_2} w^{(n-1)}(l, b, h; j, r)\ s(k_y, k_x + h; l)|_{k_y=(k+b)R} \quad (1)$$

where $s(k_y, k_x; j)$ and $\hat{s}(k_y, k_x; j)$ denote a fully sampled signal of jth coil and its estimates, $r = 1, \cdots, R$ is the index of the missing data with acceleration factor R, $s(kR, k_x; l)$ represents its acquired signal and $\hat{s}(kR + r, k_x; j)$ the estimate of unacquired signal at nth iteration. The indices b and h count blocks and columns, respectively. The weights for $r = R$ is for estimating the acquired data, which is also needed in Wiener filtering. (Step 2) Apply an adaptive Wiener filter to all the reconstructed data $\hat{s}^{(n)}$ in step 1 to reduce the error [6] using

$$\tilde{s}^{(n)}(k_y, k_x; j) = \frac{P_s^{(n)}(k_y, k_x; j)}{P_s^{(n)}(k_y, k_x; j) + \sigma_n^{2(n)}} \hat{s}^{(n)}(k_y, k_x; j), \quad (2)$$

where the error is assumed to be white-noise like, $\sigma_n^{2(n)}$ is the noise variance of the nth iteration, and $P_s(k_y, k_x; j)$ denotes the signal power spectrum. We estimate the signal power spectrum by soft thresholding the average $|\hat{s}^{(n)}(k_y, k_x; j)|^2$ in a neighborhood:

$$P_s^{(n)}(k_y, k_x; j) = \begin{cases} \frac{1}{N_s}\sum_{\text{neighborhood}} |\hat{s}^{(n)}(k_y, k_x; j)|^2 - \sigma_n^{2(n)}, & \text{if } P_{\hat{s}}^{(n)}(k_y, k_x; j) > \sigma_n^{2(n)} \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

where N_s is the size of neighborhood. The noise variance is estimated by the mean squared errors between the reconstructed and acquired data values in the ACS region. (Step 3) Find the convolution weights $w^{(n)}$ using the filtered data $\tilde{s}^{(n)}$ in the entire k-space. The above three steps are repeated until a stop condition is satisfied.

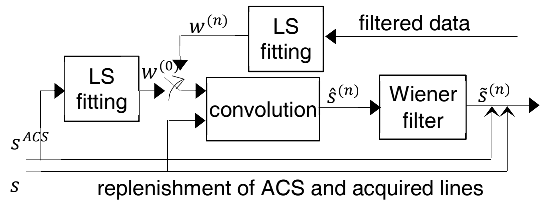

Fig. 1. Block diagram of the proposed iterative GRAPPA.

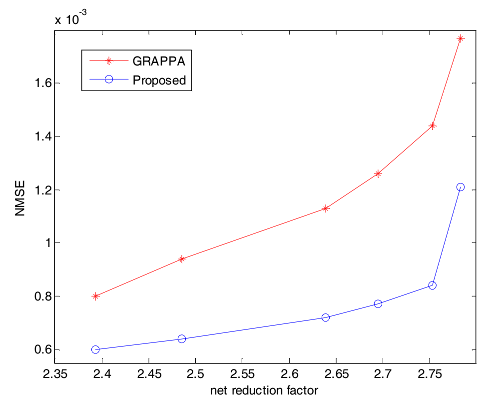

Fig. 2. NMSE comparison of two methods.

Results: We compare the performance of our method with GRAPPA using a set of in vivo axial brain data scanned on a 3T MRI scanner (GE, Waukesha, Wisconsin, USA) using an eight-channel head coil. We choose $N_s = 7 \times 7$ in (3). The number of columns and the number of blocks are $N_b = 2, N_h = 9$. The number of iterations is fixed as 10. Fig. 2 shows the NMSEs of various methods according to different net reduction factors (netR), which come from different ACS sizes from 30 to 8 at $R = 3$. We see in Fig. 2 that our method outperforms GRAPPA in NMSE over all netR. It reduces NMSEs up to maximum 58% against GRAPPA. Fig. 3 shows the reconstructions and error images of proposed method and GRAPPA with different ACS lines. The figure shows that our method yields less noise than GRAPPA and reduces aliasing artifacts.

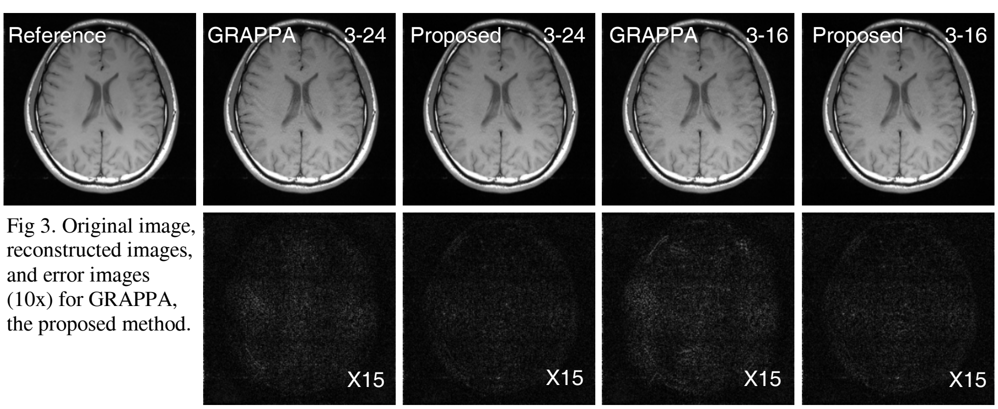

Fig 3. Original image, reconstructed images, and error images (10x) for GRAPPA, the proposed method.

Conclusion: We proposed a new method using adaptive Wiener filter. Our proposed method improves the SNR when compared to GRAPPA.

References: [1] M.A. Griswold, et al., MRM. 2002;47(6):1202-1210. [2] M. Lustig, et al., ISMRM 2007:333 [3] T. Zhao, et al., ISMRM 2007:1744 [4] S. Park, et al., MRM 2012;67(6):1721-1729. [5] Z. Wang, et al., MRM 2005;54(3):738-742. [6] J.S. Lim, Two-dimensional Signal and Image Processing, Prentice-Hall, 1990.