Design and optimization of fast imaging pulse sequences using optimal control theory.

Oleksandr Khegai[1], Jiun-Jie Wang[2], Steffen J Glaser[3], and Florian Wiesinger[4]

[1]*Healthy Aging Research Center, Chang Gung University, Taipei, Taiwan,* [2]*Department of Medical Imaging and Radiological Sciences, Chang Gung University, Taipei, Taiwan,* [3]*Department of Chemistry, Technische Universität München, Munich, Germany,* [4]*Diagnostics and Biomedical Technologies Lab, GE Global Research Europe, Munich, Germany*

Introduction: The optimal control theory (OCT) is a modern numerical approach to the dynamic optimization problem for nonlinear systems, based on the minimization of the determined cost function by searching so-called optimal path or optimal values of the selected control variables[1]. Applying OCT to the Bloch equations (which are represented by a system of bilinear equations) allows to control the dynamics of the magnetization by RF pulses and thus to design a pulse sequence, which satisfies particular predefined conditions ([2],[3],[4]). Commonly in MR research OCT was utilized for design of the individual shaped pulse that achieves after RF excitation the magnetization profile "closest" to the desired distribution for the given chemical shift (CS) offset. In this work we demonstrate the feasibility of the optimal control pulse sequence design, which allows not only for the final distribution of the magnetization, but also for the maximization and/or stabilization of the MR signal at predefined acquisition time points for certain CS offset and B_1 field inhomogeneity, typical for fast MRI. We will provide mathematical basis and show an exemplary theoretical result of the pulse sequence design, which can be used for the special case of hyperpolarized fast spin echo imaging.

Theory: The mathematical algorithm for the optimal control design of the pulse sequence is based on [4] and requires the definition of the dynamic system, control variables and appropriate cost function, which obtains the minimal value in case of the optimal path. In the following, OCT was applied to a system of magnetization vectors $\vec{M}^{i,k}$, with indices i and k representing the CS offset and B_1 inhomogeneity discretization, respectively, and corresponding angular frequencies $\vec{\omega}^{i,k}$, with components ω_x^k and ω_y^k ($\propto B_{1,x}$ and $B_{1,y}$) control variables (RF pulse) and ω_z^i off-resonance. In order to obtain stable echo train with high or predefined MR signal at acquisition time points within particular CS offset and B_1 field inhomogeneity, the following cost function J was designed:

$$J=-\sum_k\sum_i\left(\vec{M}^{i,k}(t_1)\bullet\vec{M}_{tar}\right)+\alpha\sum_k\int_{t_0}^{t_1}\left((\omega_x^k(t))^2+(\omega_y^k(t))^2\right)dt-\frac{\beta_1}{n}\sum_k\sum_i\int_{t_0}^{t_1}\sum_l\delta(t-t_l)\cdot\left|M_T^{i,k}(t)\right|dt+\frac{\beta_2}{n}\sum_k\sum_i\int_{t_0}^{t_1}\sum_l\delta(t-t_l)\cdot\left|M_T^{i,k}(t)-\exp(j\varphi_l^{i,k})\cdot\tilde{M}_T^{i,k}\right|dt$$

with t_0 and t_1 start and end time points, $\vec{M}_{tar}$ target magnetization, α, β_1, β_2 the optimization coefficients, n number of acquisitions, t_l acquisition time points, δ Dirac delta function, $M_T^{i,k}$ transversal magnetization, $\varphi_l^{i,k}$ desired phase of the acquired signal and $\tilde{M}_T^{i,k}$ mean of acquired $M_T^{i,k}$ at time points t_l $\tilde{M}_T^{i,k}=\frac{1}{n}\int_{t_0}^{t_1}\sum_l\delta(t-t_l)\cdot\left|M_T^{i,k}(t)\right|dt$. In the cost function equation the first term accounts for achieving the predefined target magnetization at the end time point t_1, second term for the deposited energy of the pulse, third term for the sum of all the acquired signals at time points t_l, and last term for the variations of MR signal. By proper choice of the coefficients, the priority of the optimization between the terms of the cost function can be set. The minus sign before some terms means that the corresponding cost should be maximized. For the case of not maximizing MR signal but keeping it at certain level, coefficient β_1 was set to zero and the mean magnetization $\tilde{M}_T$ in the last term was substituted by the desired magnetization M_{des}. Such an application is especially useful for the non-recoverable hyperpolarized MR signal, in order to keep the magnetization at the certain constant level for further acquisition time steps. The algorithm is iterative starting with arbitrary initial control variables and constant coefficient ε for the adjustment of the control variables as in [4].

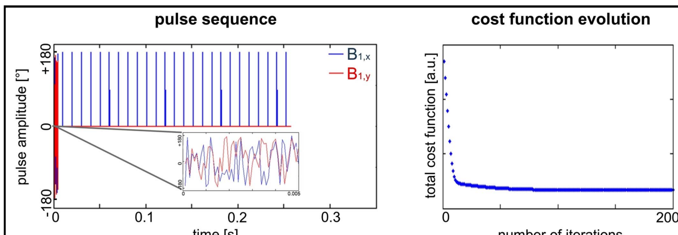

Figure 1. Pulse sequence diagram (left) of the non-CPMG pulse sequence with 50 consecutive preparation and 25 refocusing pulses. Blue and red lines show the components of the RF pulse amplitude (in degrees). The evolution and convergence of the total cost function are shown on the right.

Results: The pulse sequence optimization was exemplary performed for the design of preparation pulses for the non-CPMG pulse sequence with quadratic phase modulation of refocusing pulses as described in [5] and the desired transverse magnetization, which was chosen to be equivalent to the 45° excitation of the initial z magnetization. Preparation pulses were set as the control variables, while the following 180° refocusing pulses remained constant. The energy minimization and target magnetization terms were neglected. The CS offset of -200 to 200 Hz and the B_1 field inhomogeneity of ≈±5 % (resulted in the ±9° error of 180° refocusing pulse) with discretization steps of 20 Hz and 3° respectively were applied. The time resolution was set to 100 µs and the repetition time for signal acquisition TR=10 ms. These values correspond well to the real parameters. The optimization was performed for the case of hyperpolarized signal, i.e. high initial magnetization and neglectable relaxation processes (neglected only during the active RF pulse; for the magnetization evolution between the pulses typical values for hyperpolarized MR T_1=30 s and T_2=2 s were used). The algorithm was implemented using MATLAB. Fig. 1 shows the result of the optimization of preparation pulses (consisting of 50 consecutive hard pulses with pulse width of 100 µs) for the following echo train with 25 refocusing pulses after 200 iterations and the cost function evolution. The components of the RF pulse amplitude are expressed in degrees of the flip angle (180° ~ 5 kHz pulse amplitude). The normalized mean variation of the total transverse magnetization M_T and its M_x, M_y components at acquisition time points to the desired $M_{T,des}$, $M_{x,des}$, $M_{y,des}$ (equal to the 45° excitation MR signal distribution) are shown in Fig. 2.

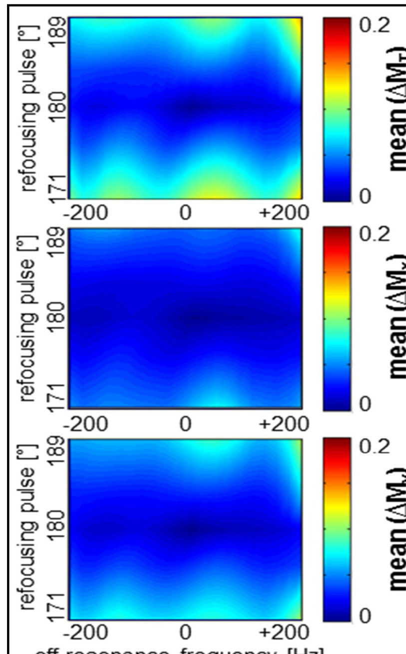

Figure 2. Normalized mean variation (in %) of the magnetization at acquisition time points to the desired magnetization for the CS offset -200 to 200 Hz and 5% B_1 field inhomogeneity. The corresponding optimized non-CPMG pulse sequence is shown in Fig.1.

Discussion: The proposed optimization method allows to optimize or design pulse sequences with following aims: i) achieving the desired target magnetization, ii) minimization of the total RF pulse energy, iii) maximization of the total MR signal and minimization of its amplitude and phase variations at acquisition time points. In other words, the result can be defined as an energetically favorable RF pulse sequence which generates stable echo train with a high total or predefined MR signal at acquisition time points and brings the magnetization to the desired final distribution. It is important to note, that the control variables (i.e. RF pulse) are allowed to be non-continuous, which is the typical case in practical MRI applications, and any RF pulse (e.g. refocusing pulse) can be set as "controllable" and therefore optimized. The algorithm does not guarantee the global minimum of the cost function and thus may lead to the non-optimal results stopping in a local minimum, especially in the case of many control or small coefficient ε. The proper choice of the variable controls and its adjustment coefficient ε helps to avoid this problem. The larger values of ε may lead to instability and non-convergence of the algorithm. It is recommended to perform the optimization several times with different initial control variables in order to achieve better results. The significant theoretical improvements and the feasibility of the method need to be confirmed in further studies by experimental results.

References: [1] B Lee et. al., Foundations of Optimal Control Theory, Wiley, 1967. [2] S Conolly et. al., IEEE Transactions on Medical Imaging 5(2):106–115, 1986. [3] D Rosenfeld et. al, MRM 36:401–409, 1996. [4] TE Skinner et. al., JMR 163:8–15, 2003. [5] P Le Roux, JMR 155:278–292, 2002.