A Lorentzian-Function-Sparsity Approach for Fast High-Dimensional Magnetic Resonance Spectroscopy

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Target Audience: Magnetic resonance spectroscopy (MRS) is a powerful tool in many fields such as medicine, pharmacy, chemistry, and biology. Moreover the high-dimensional MRS, compared with 1D spectroscopy, can provide more information such as the coupling relationship in a particular compound. However, a detrimental problem for this technology is its scanning time. This work may be beneficial to those who are interested in the significant reduction of the scanning time for high-dimensional MRS.

Purpose: The compressive high-dimensional MRS is challenging, even with the state-of-art L1-sparsity based MRS reconstruction method. For example, the reconstructed spectroscopy quality deteriorates dramatically for 25% undersampled 2D MRS data. By using the prior that the MR spectroscopy consists of Lorentzian functions[1,2], we aim to develop a robust Lorentzian-function-sparsity based spectroscopy reconstruction method for high-dimensional MRS that allows for significantly reduced scanning time with highly undersampled data, e.g., 1%.

Methods: 2D MRS with simulation data is considered in this proof-of-concept study. By using Stares-Haberkorn-Ruben method with two-dimensional cosine and sine modulated data, the pure absorption Lorentzian shape is obtained as follows: $S(\omega_1,\omega_2)_{SHR} = \gamma A_+^{(1)} A_+^{(2)} + iD_+^{(1)} A_+^{(2)}$ with $A(\omega) = \frac{1}{(\omega-\Omega)^2 T_2^2 + 1}$ and $D(\omega) = \frac{(\omega-\Omega)T_2}{(\omega-\Omega)^2 T_2^2 + 1}$, where γ control the intensity scale of each peak, T_2 is the decay parameter, and Ω is the precession frequency. Then the Lorentzian-function-sparsity based MRS reconstruction method is formulated as $\min_{\gamma,T} \| F_u X - y \|_2^2$ with $X = \sum_{i=1}^{n} \gamma_i A_i^{(1)} A_i^{(2)} + D_i^{(1)} A_i^{(2)}$. That is, the pixel-wise image reconstruction with thousands of unknowns is reduced to the parameter reconstruction with tens of unknowns[3], i.e., the center, magnitude and the shape for each peak. The simulated annealing algorithm[4] is developed to find a global minimizer for this small-scale nonlinear and nonconvex optimization problem, in which the 1D Fourier transform of the first k-line is used to identify the initial guesses of peak centers and magnitudes.

Results: For a simple single-peak case (Fig. 1), we compared FFT method, L1-sparsity method and Lorentzian-function-sparsity method with 12.5% data. FFT and L1 results contained severe artifacts while the Lorentzian method was able to provide nearly perfect reconstructed image. Then we tested a more realistic case with 7 peaks with the peak intensity scaled from 10 to 10,000 in logarithmic space(Fig. 2). Surprisingly, Lorentzian method was still able to provide almost perfect result under 1% data in less than 10 seconds. The peak signal to noise ratio(PSNR) and $||x\text{-}x_0||$ were presented to show the numerical convergence. PSNR measures the degree of peak signal restoration from background noise, which is $10\log_{10}\left(\frac{(2^n-1)^2}{MSE}\right)$, MSE is the mean squared error.

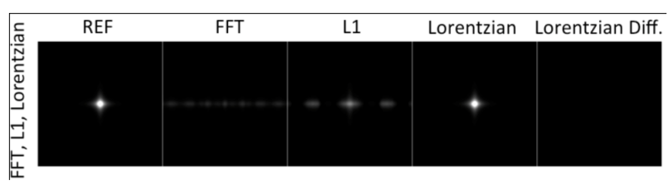

Figure 1: Single-peak Reconstruction with 25% data via FFT, L1 and Lorentzian method

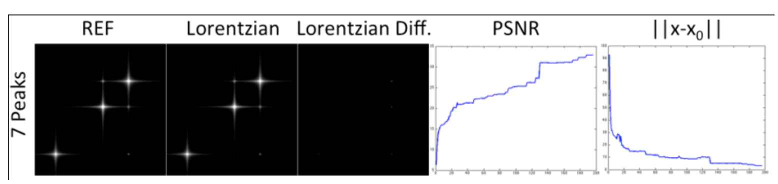

Figure 2: 7-peak Reconstruction with 1% data via Lorentzian method

Discussion and Conclusion: A new MRS reconstruction method has been proposed using the Lorentzian-function-based sparsity, with significantly reduced number of unknown variables. The new method can achieve significantly better MRS reconstruction results than FFT method or L1-based sparsity method, e.g., even with 1% k-space data (Fig.2). Its performance will be further validated using experimental data.

References: 1. Keeler. *Understanding NMR spectroscopy*. 2002. 2. Liang and Lauterbur. *Principles of Magnetic Resonance Imaging*. 2000. 3. Gao et al. *Biomed. Opt. Express*. 2014;1:1259-1277. 4. Kirkpatrick et al. *Science*. 1983;*220*:671-680.