Mathematical Tools to Define SAR Margins for Phased Array Coil In-Vivo Applications Given E-field Uncertainties

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INTRODUCTION The SAR usually defines the risk to patient safety due to the RF pulses. In a multiple transmit experiment, the final electrical field is the sum of the electrical fields produced by each transmit element, and thus, an error on the electrical field estimation is simply the sum of the error on each transmit element. The situation is more complex for the SAR due to its quadratic dependence with the electric field. The error propagation on the SAR due to an error on the estimation of the electrical field must be very carefully computed. This paper presents formulas to estimate the maximum SAR error according to some reasonable hypotheses. The results allow to compute easily margins on the SAR to ensure the safety of the patients without overconservative measures.

METHODS The electrical field and its error are generally estimated by simulations and comparison with B_1measurements [1]. These measurements however do not allow to estimate the real electrical field maps, but can give an estimation of the error in the simulation. The difference between the simulated field maps and the true field maps is expected to be smooth since they both satisfy Maxwell's equations. We will first only assume that the electrical field of channel i, called E_i has been estimated, with a random error e_i, without systematic bias (the expectation value is zero). If not, the simulated field maps can be rescaled to cancel the bias. The number of channel is n. If the input signal on channel i is s_i, the total electrical field at t is simply given by $E = \sum_1^n s_i(t)(E_i + e_i)$ so that the error on the electrical field simply is $\sum_1^n s_i(t) e_i$. The average SAR in volume V over time τ is given by: $SAR = \frac{1}{V\tau}\int_V \int_0^\tau \frac{\sigma}{2\rho} \|\sum_{i=1}^n s_i(E_i + e_i)\|^2 \, dtdv.$ When developing the square of the sum, we can verify that, even if the expectation value of e_i is zero, the average error on the SAR is strictly positive. Consequently, on average, the SAR is underestimated.

For the safety of the patient, we aim at estimating the probability for the maximum SAR, SAR_{max} to excess a given limit. For this purpose, it would be possible to compute Monte-Carlo simulations for every set of s_i waveforms, but it would require very long simulations, and if we aim at proposing one margin coefficient acceptable for every pulse, it can lead to overconservative measures. The other solution is to estimate the expectation value and the standard deviation of SAR_{max}, and consider that, the probability to exceed a given limit depends on these parameters and on the distribution. For this purpose, it is necessary to determine an upper bound for both parameters. To calculate these upper bounds, we first assume that the errors across channels are not correlated, i.e. $cov(e_i, e_j) = 0$ if $i \neq j$. The spatial correlation on the other hand, at least in small volumes, is high because the field maps are smooth [2]. The spatial correlation is assumed to be 1, i.e. the worst case scenario. The distribution of the error is assumed symmetrical: all odd moments are zero. The second moment (the variance) is supposed to be the same for every channel, every direction, and equal to σ_e^2. If it is not the case, we can consider the worst σ_e value and apply it to every field map. The fourth moment is denoted $\kappa\sigma_e^4$, where κ depends on the distribution ($\kappa = 3$ for a gaussian distribution). We will consider that the time-related shape of the RF waveform is the same for each channel i; if the shapes are different, the standard deviation is lower. With these assumptions, the expectation value of the error on the SAR, $\overline{e_{SAR}}$, and its standard deviation are

$\overline{e_{SAR}} = \frac{3}{V\tau}\int_V \frac{\sigma}{2\rho} dv \cdot \left(\int_0^\tau \sum_{i=1}^n |s_i|^2 \, dt\right)\sigma_e^2$ and $Std(e_{SAR}) = \frac{1}{V\tau}\int_V \frac{\sigma}{\rho}\int_0^\tau \sqrt{\sum_{i=1.n}\sum_{j=\{x,y,z\}} \left|s_i\left(\sum_{k=1}^n s_k E_{jk}\right)^\star\right|^2 + \frac{3}{2}\left(|\sum_{i=1.n}|s_i|^2|^2 + (\kappa - 1)\sum_{i=1.n}|s_i|^4\right)\sigma_e^2} \cdot \sigma_e dtdv.$

The risk is defined by the maximum SAR (SAR_{max}). Without knowing the location of the worst case scenario, the solution proposed is to consider a number of uncorrelated points (e.g. local SAR maxima). Among these points, we can then ignore the ones whose difference with the simulated SAR_{max} is higher than $0.1 \times Std(e_{SAR})$: their contribution to $\overline{e_{SARmax}}$ is lower than 1%. If r is the number of remaining points, the difference $\overline{e_{SARmax}} - \max(\overline{e_{SAR}})$ cannot exceed $C_r \cdot \max\left(Std(e_{SAR})\right)$, where $C_r = r\int_{-\infty}^{\infty} \frac{x}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\left(\frac{1}{2} + \frac{1}{2} erf\left(\frac{x}{\sqrt{2}}\right)\right)^{r-1} dx$. For instance, when $r = 2$ then $C_r = 0.56$. The coefficient C_r does not dramatically increase when r increases, e.g. $C_{20} = 1.87$. Consequently, the estimation of r has not to be very accurate. Finally, it can be demonstrated that the variance of e_{SARmax} is slightly lower than the maximum variance of e_{SAR}. At this point, we have an equation and upper bounds of $\overline{e_{SARmax}}$ and $Std(e_{SARmax})$ but it is almost impossible to apply it quickly to given field maps and pulses. In practice, Q matrices and VOPs [3] have been proposed to estimate the maximum SAR very quickly. Consequently, we derive a new local matrix Q_e which takes into account the error e_i, to ensure the maximum safety without overconservative measures. Considering that the error on the electrical field σ_e is significantly lower than the maximum electrical field, the distribution of error for SAR_{max} is described by $Std(e_{SARmax}) \simeq \frac{1}{V\tau}\int_V \frac{\sigma}{\rho}\int_0^\tau \sqrt{\sum_{i=1.n}\sum_{j=\{x,y,z\}}\left|s_i\left(\sum_{k=1}^n s_k E_{jk}\right)^\star\right|^2} \cdot \sigma_e dtdv$, and $\overline{e_{SARmax}} \leq C_r \cdot Std(e_{SARmax})$. Moreover, the analysis of the standard deviation shows that $Std(e_{SARmax}) \leq \frac{1}{V\tau}\int_V \frac{\sigma(x)}{\rho(x)}\int_0^\tau S' K_e S \, dtdv$, where S represents the column matrix associated to the RF waveform and $[K_e]_{ij} = \frac{\sqrt{2}}{2}\left(\|E_i\| + \|E_j\|\right)\sigma_e$. Now, considering that the probability to exceed k times the standard deviation is very unlikely (typically 0.1% of probability to exceed 3 times the standard deviation for a Gaussian distribution), the error ΔSAR_{max} in the entire sample verifies: $\Delta SAR_{max} \leq \frac{1}{V\tau}\int_V \frac{\sigma(x)}{\rho(x)}\int_0^\tau S' Q_e S \, dtdv$, where $Q_e = (k + C_r) \cdot K_e$. Finally, for a given k, and thus, for a given probability, there can exist a less conservative margin, but Q_e is close to it, and really easier to compute.

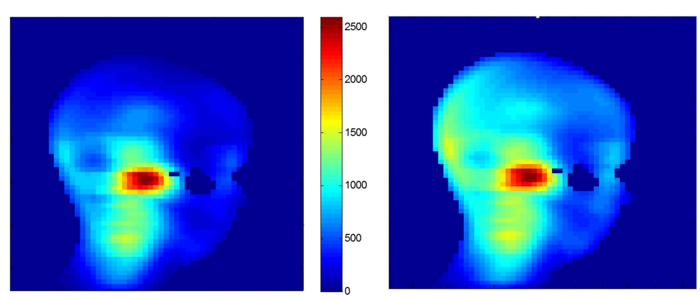

Figure 1: Average SAR before (left) and after (right) applying the correction Q_e. The standard deviation of the error is 15% of the average electrical field. Central sagittal slice of the head, arbitrary unit, same scale for both images. The maximum SAR is worth 2912 before applying Q_e and 3562 after.

As a numerical test, we simulated the electrical field in a head with the HFSS software. The volume of interest V here is a 10g volume. The Q and Q_e matrices were measured for each point, in order to estimate the SAR and the margin everywhere in the head. A set of signal s_i was determined to maximize the electrical field at the centre. The parameter r was set equal to 2, and $k = 3$. The probability to excess $Q + Q_e$ for these values is lower than 0.1 %. σ_e is defined as 15% of the average electrical field.

RESULTS Fig. 1 shows SAR_{max} before and after applying Q_e. According to the calculations and based on the assumptions presented above, the margin on SAR_{max} to ensure the safety of the patient, with a probability of 99.9%, is 22% higher than the result with Q.

DISCUSSION AND CONCLUSION Starting from the variances of the electric field uncertainties of the respective channels, estimating the expectation value and variance of the error on the maximum SAR is complex in the general case. However, we have shown that it is possible in practice to propose a reasonable overestimation of these parameters via the use of additional Q_e matrices, which can later on be combined with the construction of virtual observation points [3].

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